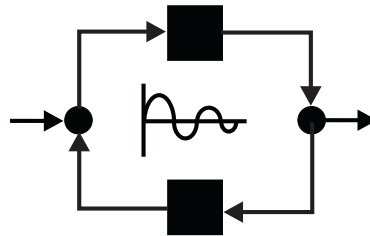


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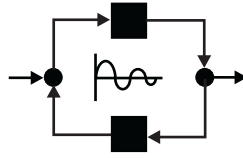
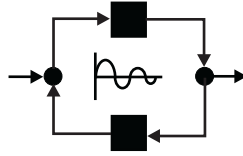


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The Control Capability Analysis and Decoupling Control of Flatness and Edge Drop for Cold Rolling Mill

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Abstract

The cold tandem rolling of metal strip presents a significant control challenge because of nonlinearities and process complexities. Flatness and edge drop are two significant parts of cold rolling strip quality. The flatness and edge drop control system is a coupled multivariable and nonlinear control system. Analyzing the coupling relationship and realizing the decoupling control between them are very important parts of achieving the high-precision control. In this paper, based on the flatness and edge drop control capabilities analysis of all the control actuators, the simplified model of flatness and edge drop is established, and the coupling relationship between flatness and edge drop is analyzed. To control the flatness and edge drop precisely in cold rolling mill, the decoupling control of flatness and edge drop is very important. Inverse decoupling control realizes the decoupling control of the original system by connecting the inverse system in series before the original system and constituting the pseudo linear composite system. It has been applied in many industrial processes successfully. Combining the neural network which has the powerful ability in nonlinear mapping and self-learning with inverse system, the neural network inverse decoupling control method is proposed to realize the decoupling control of flatness and edge drop in this paper. Simulation results represent that the flatness and edge drop control performance is improved effectively.

Keywords: Flatness; Edge Drop; Control capability; Decoupling Control; Cold Rolling Mill

1. Introduction

The cold tandem rolling of metal strip presents a significant control challenge because of nonlinearities and process complexities. Flatness and edge drop are two significant parts of cold rolling strip quality. Nowadays, Flatness control precision has satisfied the demand of users, but edge drop control level still has no breakthrough^[1]. The edge drop of strip will directly influence the side scrapping of strip and reduce the product yield^[2]. In recent years, the edge drop control

has become the concern issue in the rolling process control field. The technologies such as K-WRS (Kawasaki-Work Roll Shifting) work roll, EDC (Edge Drop Control) work roll etc. are proposed one after another^[3, 4]. However, the control system has not achieved good control effect yet. There are a lot of problems to resolve for the edge drop control. The edge drop is mainly controlled by work roll shifting. Yet, the work roll shifting inevitably influences the shape of loaded roll gap. Due to the inseparable relations between the roll gap and flatness, the flatness must be affected. The coupling relationship between the edge drop and flatness has been a hot topic^[5].

The composition control of flatness and edge drop is a coupled multivariable and nonlinear control system because of the nonlinearities and complexities in cold rolling process. So realizing the decoupling control of flatness and edge drop is one of the most important parts of achieving the high-precision control. At present, the edge drop control is compensated by adding a work roll bending compensation part during the flatness control, the control performance was improved in a certain extent^[6]. But the decoupling control of flatness and edge drop is still not realized in essence.

Inverse decoupling control realizes the decoupling control of the original system by connecting the inverse system in series before the original system and constituting the pseudo linear composite system. It has been applied in many industrial processes successfully.^[7] Combining the neural network which has the powerful ability in nonlinear mapping and self-learning^[8, 9] with inverse system, the neural network inverse decoupling control will gain the better decoupling control effect.

In this paper, the flatness and edge drop control abilities of all the control actuators are analyzed. The coupling relationship of flatness and edge drop is analyzed. The neural network inverse decoupling control method is proposed to realize the decoupling control of flatness and edge drop. Simulation results represent that the flatness and edge drop control performance is improved effectively.



The remainder of the paper is organized as follows: Section (2) focuses on the establishment of the analytic finite element model and the choice of the rolling simulation conditions. The elastic deformation of the roll and plastic deformation of the strip are considered as a whole to analyze the strip rolling process thoroughly. Section (3) emphasizes on the work roll bending, shifting and intermediate roll bending and shifting control capability analysis of the flatness and edge drop. Section (4) develops the coupling analysis between flatness and edge drop based on the results of section (3). Section (5) focuses on the decoupling control of flatness and edge drop based on the inverse system and neural network. And section (6) is conclusion of this paper.

2. The establishment of the analytic model

To deeply analyze the control characteristic and the coupling relation of the flatness and edge drop, the analytic finite element model is established in which elastic deformation of the roll and plastic deformation of the strip are considered as a whole. The simulation is for five stands 6-high tandem cold rolling mill. The roll finite element deformation model is shown as Figure1.

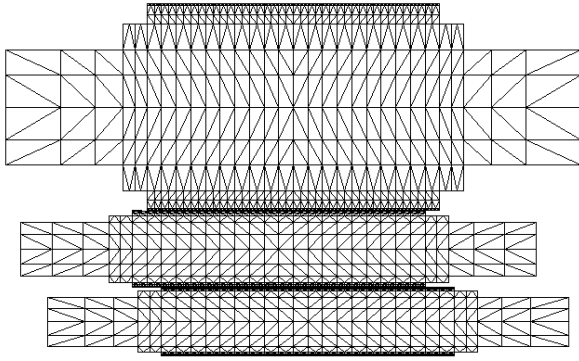


Figure1. Roll deformation model

The diameter of the backup roll is 1276mm. The diameter of the intermediate roll is 467mm. The diameter of the work roll is 426mm. The rolling velocity is 1057m/min. And the entry tension is 60Mpa, exit tension is 150 Mpa, friction coefficient is 0.05, reduction ratio is 0.05, strip width is 1100mm, strip thickness is 2.2mm. The other simulation factors about the control actuators are shown as Table 1 as follows.

Table 1 The rolling simulation conditions

Control actuators	Min	Avg	Max
Bending force of work roll (KN/ side)	-180	0	360
Bending force of intermediate roll (KN/ side)	-	0	500
Shift of work roll (mm)	-100	0	25
Shift of intermediate roll (mm)	-100	0	50

The definition of work roll shift is as follows: the shift of work roll S_w equals zero when the work roll taper corner

aims at the edge of strip; the shift is positive when the work roll taper corner enters into the edge of strip and negative when the work roll taper corner stretch out the edge of strip. The definition of intermediate roll shift is as follows: the shift of intermediate roll S_I equals zero when the intermediate roll corner aims at the edge of strip; the shift is positive when the intermediate roll corner enters into the edge of strip and negative when the intermediate roll corner stretch out the edge of strip. Figure 2 shows the definition of the work roll shift S_w and the intermediate roll shift S_I .

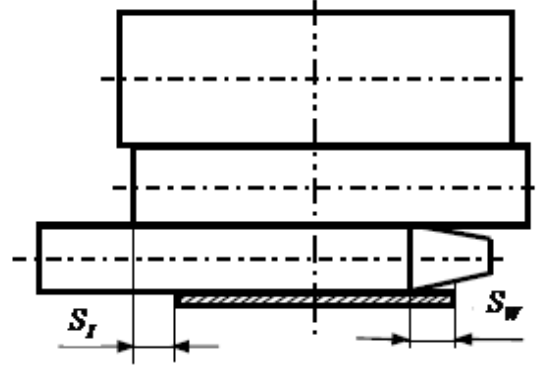


Figure 2. The definition of the roll shift

The control of flatness is actually the control of the loaded roll gap in the rolling process. The strip cross section is mainly symmetrical. The loaded roll gap can be shown as quartic polynomial as Equation1.

$$f(x) = a_0 + a_2x^2 + a_4x^4, x \in [-1, +1] \quad (1)$$

Where x is the corresponding width in view of considering the roll gap center as the original point, a_0, a_2, a_4 are coefficients.

The roll gap can be divided into quadratic part $f_2(x)$, quartic part $f_4(x)$ and constant $f_0(x)$. The constant $f_0(x)$ is expressed as Equation 2.

$$f_0(x) = f(\pm 1) = a_0 + a_2 + a_4 \quad (2)$$

The quadratic part $f_2(x)$ is expressed as Equation 3.

$$f_2(x) = C_{w2}(1 - x^2) \quad (3)$$

The curve of the quadratic part $f_2(x)$ is shown as Figure 3. It is related to the quadratic crown and the quadratic flatness of the strip.

The quartic part $f_4(x)$ is expressed as Equation 4

$$\begin{aligned} f_4(x) &= f(x) - f_0(x) - f_2(x) \\ &= a_4(x^4 - x^2) \end{aligned} \quad (4)$$



The curve of the quartic part $f_4(x)$ is shown as Figure 4. It is related to the quartic crown and the quartic flatness of the strip.

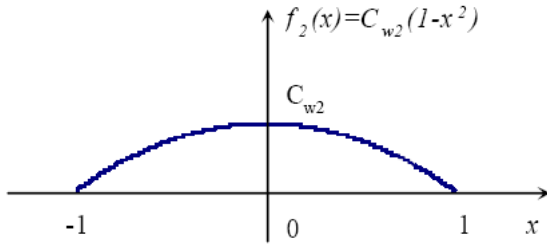


Figure3. The quadratic crown of the strip

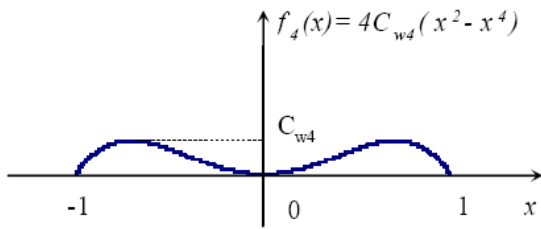


Figure4. The quartic crown of the strip

The quadratic crown C_{w2} is defined as subtracting the edge value ($x = \pm 1$) from the center value ($x = 0$) of the loaded roll gap. It can be obtained as Equation 5.

$$C_{w2} = f(0) - f(\pm 1) = -(a_2 + a_4) \quad (5)$$

The quartic crown C_{w4} is defined as subtracting the edge value ($x = \pm 1$) from the extreme value ($x = \pm\sqrt{2}/2$) of the loaded roll gap. It can be obtained as Equation 6.

$$C_{w4} = f_4\left(\pm\frac{\sqrt{2}}{2}\right) - f_4(0) = -\frac{1}{4}a_4 \quad (6)$$

So the roll gap can be expressed as Equation 7 as follows.

$$\begin{aligned} f(x) &= f_0(x) + f_2(x) + f_4(x) \\ &= a_0 + (4C_{w4} - C_{w2})x^2 - 4C_{w4}x^4 \end{aligned} \quad (7)$$

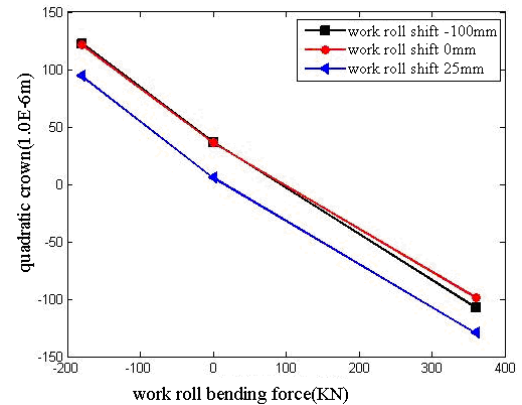
The quadratic component of the loaded roll gap corresponds to the quadratic waves of the strip. The quartic component corresponds to the quartic waves. Therefore, the control of the quadratic waves is mainly for the control of the quadratic crown, and the control of the quartic waves is mainly for the control of the quartic crown. For this reason, the flatness can be approximately expressed by the quadratic crown and the quartic crown. So the research is mainly on the quadratic crown, quartic

crown and edge drop control capabilities of control actuators in this paper.

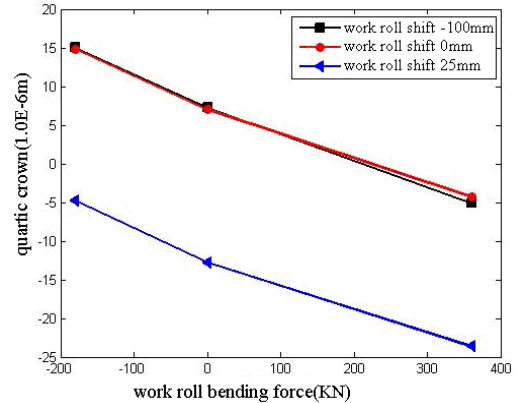
3. The control capability analysis of the flatness and edge drop

The main control actuators are the work roll bending, intermediate roll bending, work roll shifting and intermediate roll shifting on 5 stands 6-high cold tandem rolling mill. So the quadratic crown, quartic crown and edge drop control capabilities of these four control actuators are analyzed as follows.

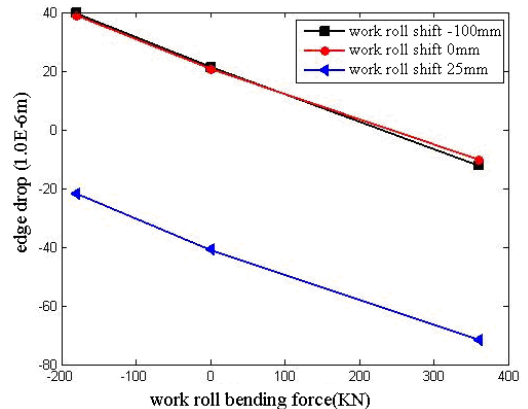
Figure5 shows the quadratic crown, quartic crown and edge drop control capabilities of the work roll bending when the intermediate roll shifting is 0mm, the intermediate roll bending force is 0KN, and the work roll shifting is -100mm, 0mm, or 25mm.



(a) the quadratic crown control capabilities



(b) the quartic crown control capabilities

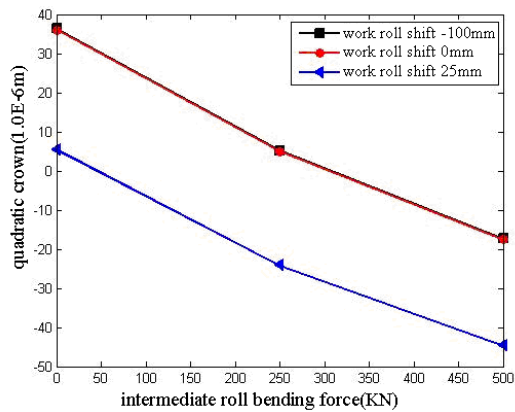


(c) edge drop control capabilities

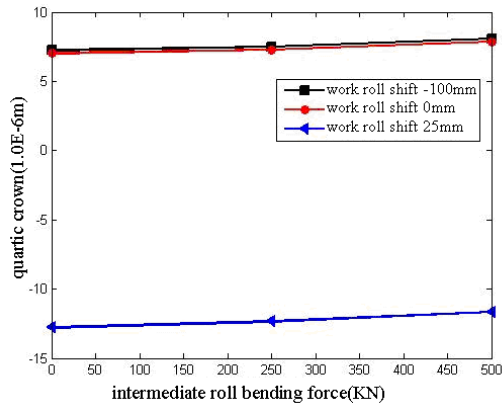
Figure5. The control capabilities of the work roll bending



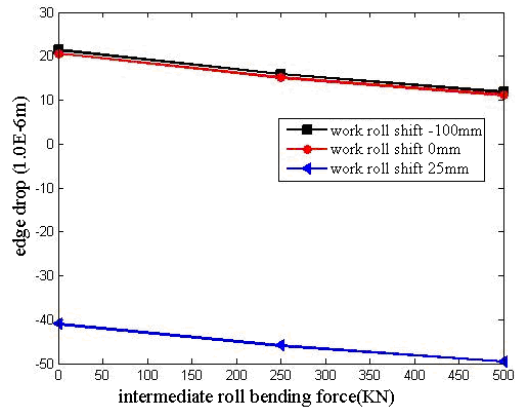
Figure (a) shows the quadratic crown curve changing with the work roll bending. Figure (b) shows the quartic crown curve changing with the work roll bending. Figure (c) shows the edge drop curve changing with the work roll bending. The results represent that the quadratic crown, the quartic crown and edge drop all decrease with the increasing work roll bending force and the changes are nearly linear. But the quadratic crown control capability of work roll bending is most effective. The control scope of the quadratic crown is about 220 μm under the work roll bending control. The control scope of the edge drop is about 50 μm . And the quartic crown control ability of the work roll bending is about 20 μm . So the work roll bending is mainly used to control the quadratic crown. It is also a way to control edge drop.



(a) the quadratic crown control capabilities



(b) the quartic crown control capabilities



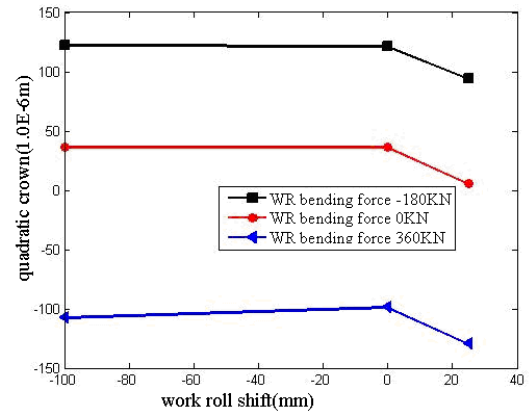
(c) edge drop control capabilities

Figure6. The control capabilities of the intermediate roll bending

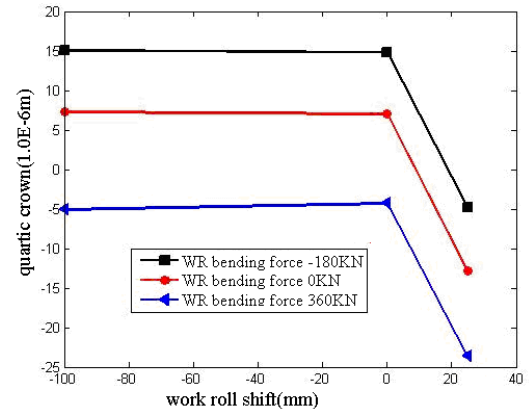
Figure6 shows the quadratic crown, quartic crown and edge drop control capabilities of the intermediate roll bending when the intermediate roll shifting is 0mm, work roll bending force is 0KN, and work roll shifting is -100mm, 0mm, or 25mm.

Figure (a) shows the quadratic crown curve changing with the intermediate roll bending. Figure (b) shows the quartic crown curve changing with the intermediate roll bending. Figure (c) shows the edge drop curve changing with the intermediate roll bending. The results represent that the quadratic crown and edge drop all decrease with the increasing intermediate roll bending force and the changes are nearly linear. But the quartic crown increases with the increasing intermediate roll bending force and the change is nearly linear. The quadratic crown control capability of intermediate roll bending is about 50 μm under the intermediate roll bending control. And the quartic crown control orientation of the intermediate roll bending is same as the work roll bending. The control scope of the edge drop is about 10 μm . So the intermediate roll bending is seldom used for the control of the edge drop. And the quartic crown control ability of the intermediate roll bending is about 20 μm . And the quartic crown control orientation of the intermediate roll bending is opposite to the work roll bending. So it is important to consider the control strategy of the work roll bending and the intermediate roll bending to control the quadratic and quartic crown.

Figure7 shows the quadratic crown, quartic crown and edge drop control capabilities of the work roll shifting when the intermediate roll shifting is 0mm, intermediate roll bending force is 0KN, and work roll bending is -180KN, 0KN, or 360KN.



(a) the quadratic crown control capabilities



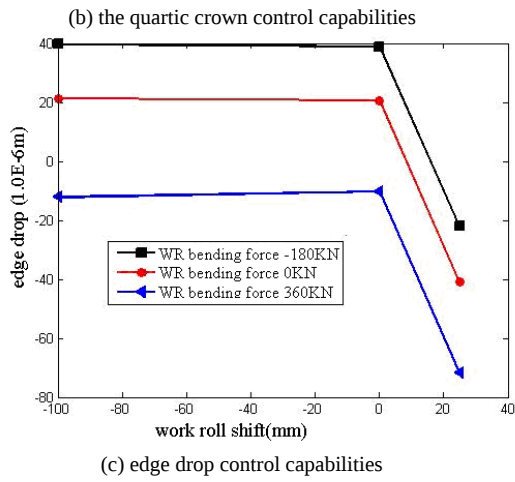


Figure7. The control capabilities of the work roll shifting

Figure (a) shows the quadratic crown curve changing with the work roll shifting. Figure (b) shows the quartic crown curve changing with the work roll shifting. Figure (c) shows the edge drop curve changing with the work roll shifting. The results represent that the quadratic crown, the quartic crown and edge drop all change little with the increasing work roll shifting when work roll shifting value is smaller than 0mm, but the quadratic crown, the quartic crown and edge drop all decrease with the increasing work roll shifting when work roll shifting value is greater than 0mm. So the edge drop is controlled effectively when the work roll shifting is positive. The edge drop can decrease 60um when the work roll shifting is from 0mm to 25mm. So the work roll shifting is the most effective way to control edge drop. At the same time, the quadratic crown decreases about 35um and the quartic crown decreases about 20um. So the coupling relation exists between the edge drop and flatness control. Figure8 shows the quadratic crown, quartic crown and edge drop control capabilities of the intermediate roll shifting when the work roll shifting is 0mm, intermediate roll bending force is 0KN, and work roll bending is -180KN, 0KN, or 360KN.

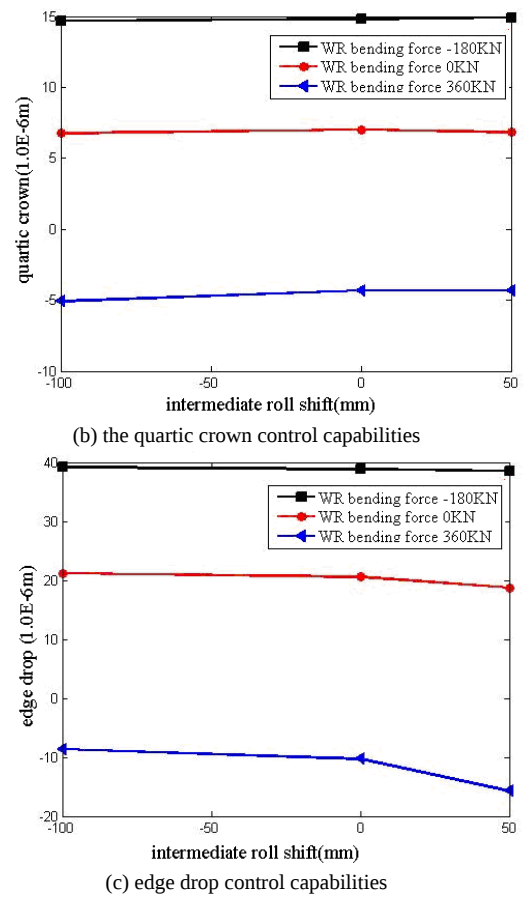
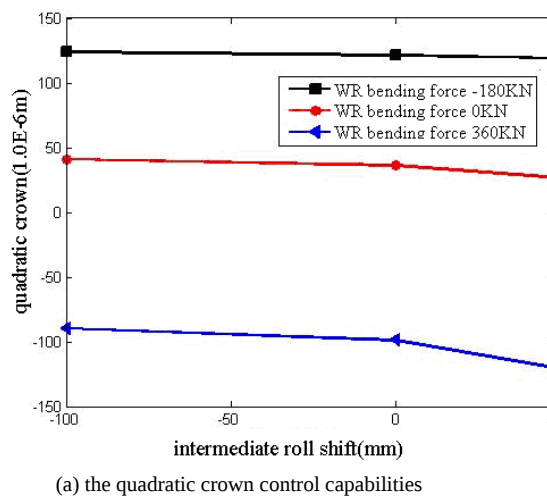


Figure8. The control capabilities of the intermediate roll shifting

Figure (a) shows the quadratic crown curve changing with the intermediate roll shifting. Figure (b) shows the quartic crown curve changing with the intermediate roll shifting. Figure (c) shows the edge drop curve changing with the intermediate roll shifting. The results represent that the quadratic crown and edge drop both decrease slowly with the increasing intermediate roll shifting. But the quartic crown increases slowly with the increasing intermediate roll shifting. So in the cold rolling control system, the intermediate roll shifting is usually set in the preset control system and seldom changed in the feedback control.

In the real rolling process, four control actuators are used according to the different control strategy to meet different demands for kinds of cold rolling mills. The main purpose is to develop the control ability of each control actuator to the most effective extent. The analysis of the control capability of control actuators is very important for the control strategy. In view of above analysis, the quadratic crown control capability sequence of the actuators is: the capability of the work roll bending > the capability of the work roll shifting > the capability of the intermediate roll bending > the capability of the intermediate roll shifting; the quartic crown control capability sequence of the actuators is: the capability of the work roll bending > the capability of the intermediate roll bending > the capability of the intermediate roll shifting; the edge drop control capability sequence of the actuators is: the capability of the work roll shifting > the capability of the work roll bending > the capability of the



intermediate roll bending > the capability of the intermediate roll shifting.

4. The coupling analysis between flatness and edge drop

The flatness is approximately expressed by the quadratic crown and the quartic crown. the quadratic crown is more larger than the quartic crown, thus, the coupling model of the flatness and edge drop can be simplified as the coupling model of the quadratic crown and edge drop.

From the control capability of the actuators, it can be seen that the strongest control capability of the edge drop is work roll shifting, secondly is work roll bending. The control of the intermediate roll shifting and bending can be ignored. The strongest control capability of the quadratic crown is work roll bending, secondly is work roll shifting. The control of the intermediate roll shifting and bending can be ignored. Hence, the coupling model of the flatness and edge drop can be simplified as the coupling model of the quadratic crown and edge drop controlled by the work roll bending and work roll shifting. In the actual rolling process, the quadratic crown is mainly controlled by work roll bending. Edge drop is mainly controlled by work roll shifting. But edge drop is influenced during controlling the quadratic crown by work roll bending and coupling relation is existed. Equally, the quadratic crown is influenced during controlling edge drop by work roll shifting.

The flatness and edge drop control capability of each control actuator can be defined as induced variation of the quadratic crown, the quartic crown and edge drop per change of the control actuator. The flatness and edge drop control capabilities of the work roll bending and the work roll shifting are as table 2 shown as follows.

Table2 The flatness and edge drop control capability of the actuators

Control actuators	Work roll bending	Work roll shifting
the quadratic crown	-0.40736	-0.24779
Edge drop	-0.09094	-0.49904

In this paper, the coupling coefficient k_{e2} is defined as the influence for the quadratic crown during controlling edge drop by work roll shifting as Equation 8.

$$k_{e2} = \frac{K_{C_{W2}S_W}}{K_{C_eS_W}} \quad (8)$$

Where $K_{C_{W2}S_W}$ is the quadratic crown control capability of the work roll shifting; $K_{C_eS_W}$ is the edge drop control capability of the work roll shifting.

The coupling coefficient k_{2e} is defined as the influence for the edge drop during controlling the quadratic crown by work roll bending as Equation 9.

$$k_{2e} = \frac{K_{C_eF_W}}{K_{C_{W2}F_W}} \quad (9)$$

Where $K_{C_eF_W}$ is the edge drop control capability of the work roll bending; $K_{C_{W2}F_W}$ is the quadratic crown control capability of the work roll bending.

As a result, $k_{e2} = 0.496533$ and $k_{2e} = 0.0223242$, the coupling relation is strong during controlling the quadratic crown by work roll bending and controlling the edge drop by work roll shifting. Especially the influence for the quadratic crown during controlling edge drop by work roll shifting is very large. The control quality characteristic is decreased, so the flatness and edge drop need to be decoupled in the control process.

5. The Decoupling Control of Flatness and Edge Drop

Inverse system can achieve the representation from the output to the input of the original system. Inverse decoupling control realizes the decoupling control of the original system by connecting the inverse system in series before the original system and constituting the pseudo linear composite system.

If the original system has no precision model, the analytic inverse system can not be established. The tandem rolling of cold metal strip presents a significant control challenge because of nonlinearities and process complexities. Due to the powerful ability in nonlinear mapping and self-learning of the neural network, the flatness and edge drop of the strip control model can be established by the neural network. So the neural network inverse decoupling control combining the neural network nonlinear mapping capability with inverse system decoupling control will gain the better decoupling control effect^[10, 11]. Neural network inverse decoupling control method achieves the dynamic function of the inverse system by adding several delay factors or integrators to the static neural network. The static neural network approximates the static nonlinear function and the delay factor or integrators reflect the dynamic characters of the system. The whole system can be changed to the decoupled normalized pseudo linear composite system. It has the linear transmit relation by connecting the neural network inverse system in series before the original system.

Flatness and edge drop control actuators model can be simplified to second order dynamic system because the work roll bending and work roll shift are controlled by the hydraulic cylinder and servo valve^[12]. The inputs of the inverse system are the quadratic crown and edge drop which are the outputs of the original system and first order, second order derivative of quadratic crown and edge drop. The outputs of the inverse system are work roll bending and work roll shift which are the inputs of the original system. The pseudo linear composite system is established by connecting the neural network inverse system and original system in series. The system is shown as Figure 9. The inputs of this system are the first order, second order derivatives of the edge drop and the quadratic crown which are connected to static neural network inverse system by four integrators. The outputs of the edge drop and the quadratic crown can be obtained by neural network inverse system and original system. So the flatness and edge drop has been decoupled to two independent systems.



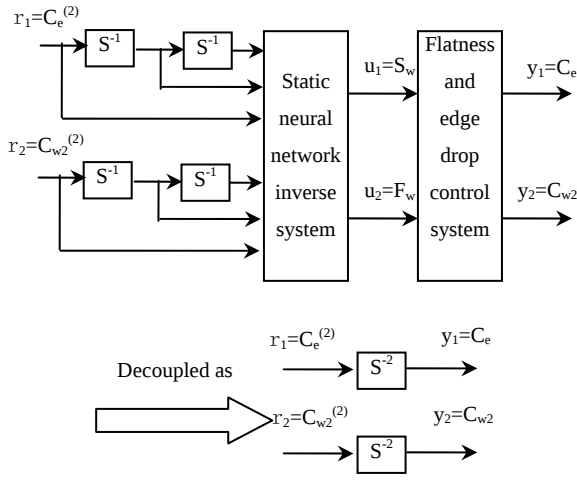


Figure 9. The pseudo linear composite system

The pseudo linear composite system is equivalent to two linear integral subsystems which are two 2-order subsystem of quadratic crown and edge drop. Then a close-loop controller can be designed by using the linear system theorem for the pseudo linear composite system. The whole control principle of the flatness and edge drop neural network inverse decoupling system is shown as Figure 10. The control system simulation is accomplished by the MATLAB/SIMULINK.

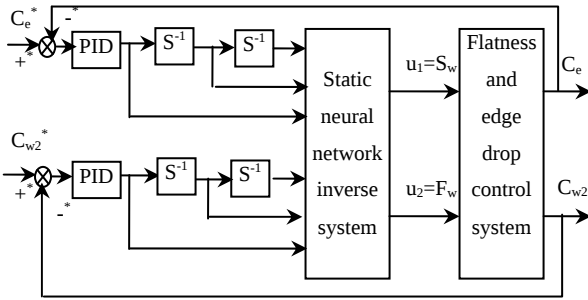
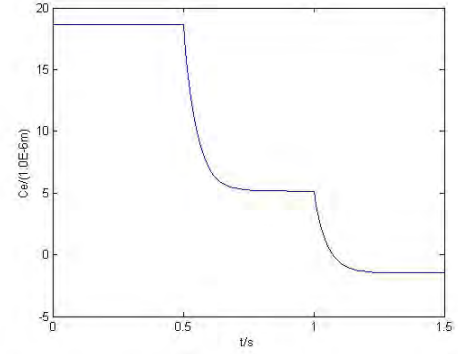
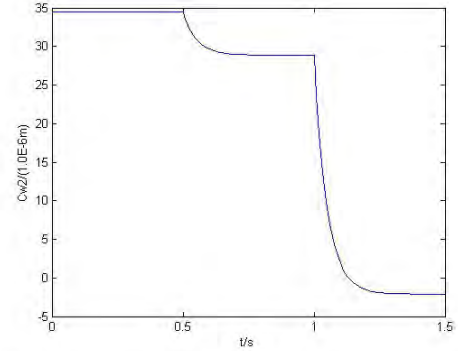


Figure10. The decoupling control of the flatness and edge drop

The edge drop and flatness control curve of the original system without decoupling control is shown as Figure 11. It can be seen from Figure11 that work roll shift value changes from 0mm to 5mm at 0.5 second during controlling edge drop by work roll shift in the original system. The edge drop changes from 19 μ m to 5 μ m, the quadratic crown also changes from 35 μ m to 29 μ m. During controlling the quadratic crown by work roll bending, work roll bending value changes from 0KN to 80KN at 1 second. It can be seen that the quadratic crown changes from 29 μ m to -2 μ m, the edge drop also changes from 5 μ m to -2 μ m. Simulation results show that the strong coupling relation exists between the edge drop and the quadratic crown in the original system. So if the decoupling control is not considered, the control effect is not ideal and the strip quality is not satisfactory.



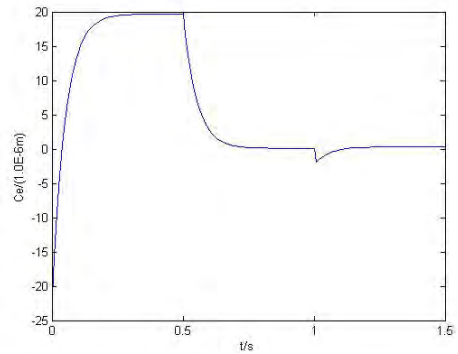
(a) the edge drop output



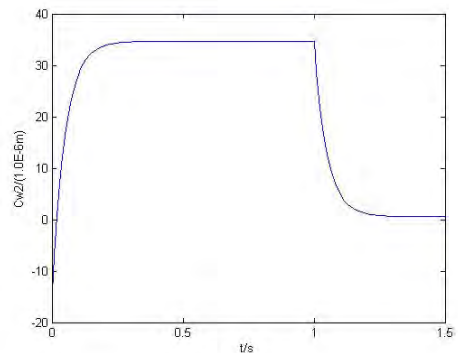
(b) the quadratic crown output

Figure 11. The output of the original system

The edge drop and flatness control curve of the inverse decoupling system is as Figure12 shown.



(a) edge drop output



(b) the quadratic crown output

Figure 12. The output of the decoupling system



It can be seen from Figure12 that the edge drop changes from 20um to 0um at 0.5 second during controlling edge drop by work roll shift in the pseudo linear composite system, but the quadratic crown nearly has no change. And during controlling the quadratic crown by work roll bending, the quadratic crown changes from 35um to 0um at 1 second, the edge drop changes little, less than 3um. Simulation results show that the neural network inverse system decoupling system has a good dynamic decoupling effect between the edge drop and the quadratic crown.

6. Conclusion

The flatness and edge drop control capabilities of the actuators for the cold tandem rolling mill are analyzed in this paper. The simplified model of the flatness and edge drop is established and the coupling relation between flatness and edge drop is quantitatively analyzed. From the analysis results, the edge drop is influenced when flatness is controlled by work roll bending and flatness is influenced when edge drop is controlled by work roll shifting. The strong coupling relation exists between the flatness and edge drop. The neural network inverse decoupling system of the flatness and edge drop is established. The decoupling control is completed by combining the inverse system with the original system into pseudo linear composite system. Simulation results show that the neural network inverse decoupling system has a good dynamic decoupling effect between edge drop and flatness control.

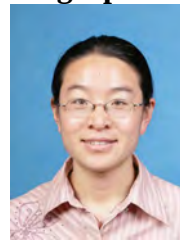
7. Acknowledgements

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8. References

- [1] Yang G H, Zhang J, Cao J G. Edge Control Technology of 4-hi and 6-hi Tandem Cold Rolling Mills[J],Metallurgical Equipment, 178(5), pp1-pp5, 2009.
- [2] Tateno J, Kenmochi K, Yarita I. Controlling Edge Drop by Tapered -Crown Work Roll Shifting Mill and Work Roll Crossing Mill in Cold Strip Rolling. Journal of the JSTP[J], (7).pp653-pp656.1999
- [3] Lu H T, Cao J G, Zhang J. Edge drop control of a taper roll during continuous cold rolling[J]. Journal of University of Science and Technology Beijing, 28(8), pp774-pp777,2006.
- [4] Zhou X M, Zhang Q D, Wu P. The Summary of the Strip Edge Drop Control Technology[J]. ShangHai Metals.,29(1), pp21-pp24,2007.
- [5] ZHOU X M, WU Y F, MAO YJ. The Research on the Coupling Relation between Flatness and Edge Drop for Tandem Cold Rolling Mill. Advanced Materials Research .572(8),pp104-pp109,2008.
- [6] Zhou X M, Zhang Q D, Wang C S, et al. Edge Drop and Flatness Decoupling Control on UCMW Cold Mill[J]. Iron and Steel,42(5), pp55-pp57,2007.
- [7] Jiancheng Fang and Yuan Ren. Decoupling Control of Magnetically Suspended Rotor System in Control Moment Gyros Based on an Inverse System Method. IEEE/ASME Transactions on Mechatronics,17(6). pp1133 – pp1144.2012
- [8] Y.V.Siva Reddy,M.Vijayakumar and T.Brahmananda Reddy, Direct Torque Control of Induction Motor Using Sophisticated Lookup Tables Based on Neural Networks , The International Journal of Artificial Intelligence and Machine Learning[J],7(1). pp9-pp15, 2007.
- [9] S.G. Kadwane and Amit kumar and B.M. Karan and T Ghose. Online Trained Simulation And DSP Implementation Of Dynamic Back Propagation Neural Network For Buck Converter. ICGST International Journal on Automatic Control and Systems Engineering [J].6(1). pp27-pp34.2006
- [10] Y. Z. Hong, H. Q. Zhu and Q. H. Wu. Dynamic Decoupling Control of AC-DC Hybrid Magnetic Bearing Based on Neural Network Inverse Method, International Conference on Electrical Machines and Systems, pp3940-pp3943. 2008.
- [11] Y. X. Song, J. L. Ma and H. X. Zhang. The Decoupling Control of Induction Motor Based on Artificial Neural Network Inverse System Method, International Conference on Electrical and Control Engineering, pp2287-pp2290.2010.
- [12] P. Peng: Research on Coupling Relations and Decoupling Control for Wide Strip Tandem Cold Mill. (Ph.D., University of Science and Technology Beijing, China 2008), pp.58.2008

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SIMULATION OF PARTICLE TRAJECTORIES IN A 3-PHASE COMMON ENCLOSURE GAS INSULATED BUSDUCT WITH EPOXY COATED ELECTRODES

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Abstract

In this context of changing and challenging market requirements, Gas Insulated Substation (GIS) has found a broad range of applications in power systems for more than two decades because of its high reliability, easy maintenance and small ground space requirement etc. SF₆ has been of considerable technological interest as an insulation medium in GIS because of its superior insulating properties, high dielectric strength at relatively low pressure and its thermal and chemical stability. SF₆ is generally found to be very sensitive to field perturbations such as those caused by conductor surface imperfections and by conducting particle contaminants. The purpose of this paper is to develop techniques, which will formulate the basic equations that will govern the movement of metallic particles like aluminum, copper in a coated as well as uncoated busduct.

Keywords: Gas Insulated Substation, Busduct, Particle, Epoxy

1. Introduction

In this context of changing and challenging market requirements, Gas Insulated Substation has found a broad range of applications in power systems for more than two decades because of its high reliability, easy maintenance and small ground space requirement etc. SF₆ has been of considerable technological interest as an insulation medium in GIS because of its superior insulating properties, high dielectric strength at relatively low pressure and its thermal and chemical stability. In uniform field the dielectric strength of pure SF₆ is approximately three times that of nitrogen in same conditions. Although, GIS has been in operation for several years, some of the problems need full attention. These problems include generation of over voltages during switching operations like enclosure faults and particle contamination. Sulphur

Hexafluoride (SF₆) is generally found to be very sensitive to field perturbations such as those caused by conductor surface imperfections and by conducting particle contaminants. A study of CIGRE group suggests that 20% of failures in Gas Insulated Substations (GIS) are due to the existence of various metallic contaminations in the form of loose particles.[1] However, in non uniform fields it depends on many factors such as electrode geometry, voltage wave shape, polarity, gas pressure etc. although in GIS, a highly non- uniform field would be generally avoided, such divergent fields can occasionally exist due to electrode surface roughness or dust or conducting particles between electrodes. As a result of this, breakdown could occur due to local field enhancement.

Several authors conducted experiments on insulating particles. Insulating particles are found to have little effect on the dielectric behaviour of the gases [1-8]. However the presence of atmospheric dust containing conducting particles, especially on the cathode, reduces the breakdown voltage.

Conducting particles placed in a uniform ac field lift-off at a certain voltage. As the voltage is raised, the particles assume a bouncing state reaching a height determined by the applied voltage. With a further increase in voltage, the bounce height and the corona current increase until break down occurs [9]. The lift off voltage is independent of the pressure of gas. After the onset of bouncing, the offset voltage is approximately 30% lower than the lift-off voltage.

Several methods of conducting particle control and de-activation are given. [2] Some of them are:

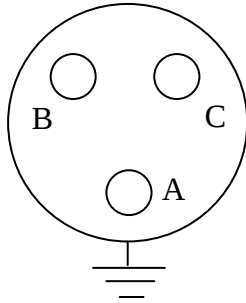
- a. Electrostatic trapping
- b. Use of adhesive coatings to immobilize the particles
- c. Discharging of conducting particles through radiation, and
- d. Coating conducting particles with insulating films[3]



The work reported in this paper deals with the movement of metallic particle in 3-phase common enclosure Gas Insulated Busduct with uncoated and coated enclosure. In order to determine the axial movement in an uncoated enclosure system, the Monte-Carlo technique has been adopted in conjunction with motion equation. The present paper deals with the computer simulation of particle movement in 3-phase common enclosure GIB with coated as well as uncoated system. The specific work reported deals with the charge acquired by the particle due to macroscopic field at the tip of the particle, the force exerted by the field on the particle, drag due to viscosity of the gas and random behaviour during the movement. Wire like particles of aluminium and copper of a fixed geometry in a 3-phase busduct have been considered. The particle movement has been calculated in uncoated and coated busducts. The movement patterns at various power frequency voltages have been obtained. Monte-Carlo technique has been adopted for determining the axial movement of the particle. It has been assumed that at every time step the particle comes out from its location at an angle less than 1° .

2. Modeling Technique

The Figures 1 & 2 show a typical horizontal three phase busduct with and without dielectric coating respectively. The enclosures are filled with SF₆ gas at high pressure. A particle is assumed to be at rest at the enclosure surface, just beneath the busbar A, until a voltage sufficient enough to lift the particle and move in the field is applied.



A, B, C are the conductors
Figure 1. A typical 3-phase common enclosure GIB

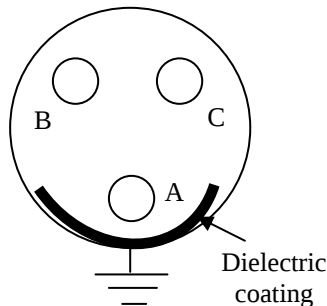


Figure 2. Typical 3-phase common enclosure GIB with dielectric coating

After acquiring an appropriate charge in the field, the particle lifts and begins to move in the direction of

field having overcome the forces due to its own weight and air drag. The simulation considers several parameters e.g. the macroscopic field at the surface of the particle, its weight, Reynold's number, coefficient of restitution on its impact to both enclosures and viscosity of the gas. During return flight, a new charge on the particle is assigned based on the instantaneous electric field.

3. Theoretical Study

Many authors [1-4] have suggested solutions for the motion of a sphere or a wire like metallic particle in an isolated busduct system.

The gravitational force F_g acting on a particle of mass 'm', with 'g' the acceleration due to gravity, is given by

$$F_g = mg \quad \dots (1)$$

The charge acquired by a vertical wire particle in contact with a bare enclosure can be expressed as [5]:

$$Q_{net} = \frac{\pi \epsilon_0 l^2 E(t_0)}{\ln\left(\frac{2l}{r}\right) - 1} \quad \dots (2)$$

where Q_{net} is the charge on the particle until the next impact with the enclosure, l is the particle length, r is the particle radius, $E(t_0)$ is the ambient electrical field at $t = t_0$.

The charge carried by the particle between two impacts has been considered constant in the simulations. This is a reasonable assumption when the applied voltage is low or when the gas pressure is high. The electrostatic force

$$F_e = KQ_{net}E \quad \dots (3)$$

where K is the correction factor less than unity, Q_{net} is the particle charge and E is ambient electric field.

The expression for drag force is given by:

$$F_d = (t)\pi r(6\mu K_d(\dot{y}) + 2.656[\mu\rho_g l\dot{y}(t)]^{0.5}) \quad \dots (4)$$

where μ is the viscosity of the fluid (SF₆: 15.5×10^{-6} kg/m.s at 20°C), r is the particle radius, ρ_g is the gas density, l is the particle length, $K_d(\dot{y})$ is a drag coefficient.

The motion equation is given by

$$\frac{md^2y}{dt^2} = F_e - mg - F_d \quad (5)$$

Where, y is the direction of motion and F_d is the drag force. The direction of the drag force is always opposed to the direction of motion. For laminar flow the drag force component around the hemispherical ends of the particle is due to shock and skin friction.

Very limited publication is available for the movement of particle in a three-phase busduct, however the equation of motion is considered to be same as that of an isolated phase busduct.



Monte-Carlo technique

Solving the above equation gives the radial movement of moving metallic particle. But the particle geometric configuration of the tips is generally uneven and the movement of the particle is not unidirectional. So, the particle also moves in axial direction because of particle surface roughness at the tips and randomness of this axial movement can be reasonably simulated by using Monte-Carlo technique. For computing axial movement of metallic particle, it is assumed that the particle comes out from its location at an angle less than θ . At every step of movement, A new rectangular random number between 0 and 1 is generated at every step of movement. Accordingly θ is modified by multiplying it with random number and this fixes the position of particle at every time step for determining the new location of moving metallic particle in terms of axial and radial movements. In the next step again new position is computed using particle motion equation with newly generated random angles as described above. However it is be noted that the usage of Monte-Carlo technique yields better axial movements while the radial movements remain the same.[4,10]

4. Simulation of electric field in 3-phase busduct with and without coating

The charge acquired by the horizontal wire particles in contact with a bare enclosure and coated enclosure can be expressed as given by Srivastava and Anis [2]. The electric field in 3-phase common enclosure GIB electrode system at the position of the particle can be written as

$$E(t) = E_1(t) + E_2(t) + E_3(t) \dots (6)$$

Where, $E(t)$ is the resultant field in vertical direction due to the field of three conductors on the surface of the particle at the enclosure.

$E_1(t)$, $E_2(t)$ and $E_3(t)$ are the components of the electrical field in vertical direction. The gravitational force and drag forces are considered as described by several authors.

5. Simulation of Particle Motion

Computer simulations of the motion of metallic wire particles were carried out on GIB of 64mm inner diameter for each enclosure and 500mm outer diameter with various voltages 245KV, 300 KV, 400KV applied to inner conductors with 120° phase difference.

A conducting particle motion, in an external electric field will be subjected to a collective influence of several forces. The forces may be divided into

Electrostatic Force (Fe)

Gravitational Force (mg)

Drag Force (Fd)

Software was developed in C language considering the above equations and was used for all simulation studies.

6. Results and Discussions

Figure 3 shows the movement of aluminium particle in radial direction for an applied voltage of 400KV rms. During its movement it makes several impacts with the enclosure. The highest displacement in radial direction during its upward journey is simulated to be 65mm. As the applied voltage increases the maximum radial movement also increases as given in Table 1. However, it is noticed that even up to a voltage of 100kv, the particle could not bridge the gap. Further calculations may reveal the limiting voltage to enable the particle to reach the high voltage conductor. A graphical representation of radial movement in relation to axial movement is given by Monte-carlo technique as shown in Figure 4. The movement of copper particle was determined for 400kv with similar parameters as above and found to have a maximum movement of 18mm in radial direction as shown in Figure 5. The movements are also calculated for other voltages.

The movement of copper particles is also given in Table 1. It is noticed that the movement of copper particle is far less than aluminium particle of identical size. This is expected due to higher density of copper particle. The axial and radial movements of aluminium and copper particles are calculated using Monte-Carlo technique for three voltages i.e. 245KV, 300KV and 400KV with a solid angle of 1° . It is significant to note that for all the cases considered, there is no change in maximum radial movement, even when Monte-Carlo method is applied. A relatively high value of axial movement is achieved with the random angle of 1° . As expected the axial movement of copper particle is lower than the aluminium. The movement of aluminium and copper particles for a coated enclosure surface is shown in Figures 6 and 7 respectively. It can be seen that the movement has been far less than that of an uncoated system.

Table 1 : Axial and Radial movement of aluminium and copper particles (Simulation time : 1 sec)

Voltage	Type	Max. Radial Movement (mm)	Monte-Carlo ($\theta=1^\circ$)	
			Radial (mm)	Axial (mm)
245KV	Al	28	28	320
	Cu	13	13	280
300KV	Al	32	32	340
	Cu	14	14	300
400KV	Al	65	65	600
	Cu	18	18	340



Table 2: Axial and Radial movement of aluminium and copper particles with coated electrodes, thickness 50 μm (Simulation time: 1 sec)

Voltage	Type	Max. Radial Movement (mm)	Monte-Carlo ($\theta=1^\circ$)	
			Radial (mm)	Axial (mm)
245KV	Al Cu	0.001 0.0001	0.001 0.0001	100 90
300KV	Al Cu	0.02 0.002	0.02 0.002	210 183
400KV	Al Cu	0.4 0.014	0.4 0.014	195 190

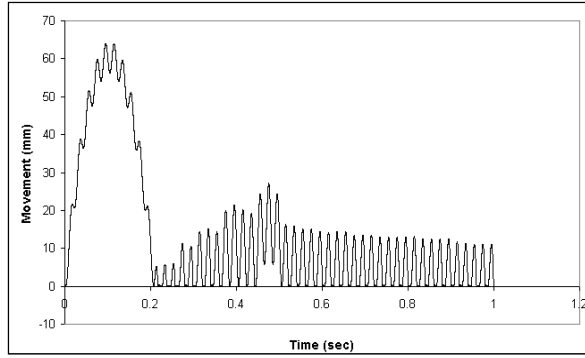


Figure 3. Particle Movement in 3-phase GIB (400KV/Al / 0.1mm / 10mm)

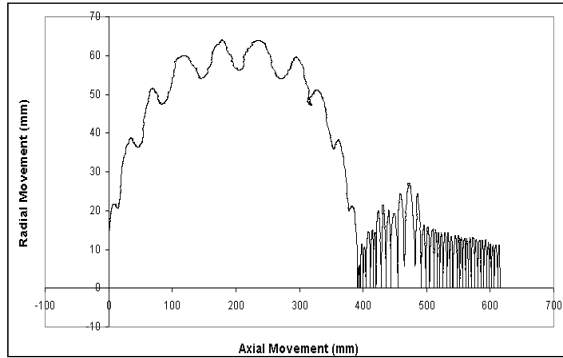


Figure 4. Particle Movement in 3-phase GIB (400KV/Al / 0.1mm / 10mm / 1° Monte-Carlo technique)

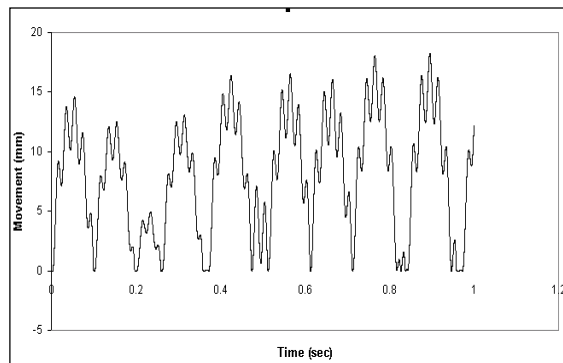


Figure 5 : Particle Movement in 3-phase GIB (400KV/Cu / 0.1mm / 10mm)

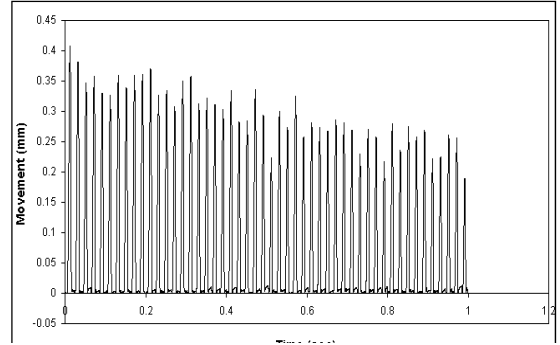


Figure 6 : particle Movement in 3-phase GIB on coated electrodes (400KV/Al / 0.1mm / 10mm / 50 micro meter)

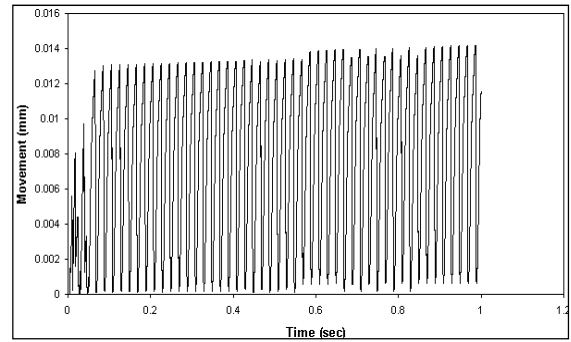


Figure 7 : particle Movement in 3-phase GIB on coated electrodes (400KV/Cu / 0.1mm / 10mm / 50 micro meter)

7. Conclusions

A model has been formulated to simulate the movement of wire like particle in 3-phase common enclosure GIB on bare as well as coated electrodes. The results obtained are presented and analyzed. Monte-carlo simulation is also adopted to determine axial as well as radial movements of particle in the busduct. Distance traveled in the radial direction is found to be same with or without Monte-carlo simulation. The displacement on coated electrodes are found to be very less compared to uncoated electrode system.

8. Acknowledgements

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9. References

- [1] H Cookson, PC Bolin, HC Doepken, RE Wootton, CM Cooke, JG Trump; "Recent Research in the United States on the Effect of Particle Contamination Reducing the Breakdown Voltage in Compressed Gas Insulated System"; Int. Conf. On Large High Voltage System; Paris, 1976.
- [2] H.Anis, KD Srivastava; "Breakdown Characteristics of Dielectric Coated Electrodes in Sulphur Hexafluoride Gas with Particle Contamination", Sixth International Symposium on High Voltage Engineering No. 32.06, New Orleans, LA, USA, 1989.
- [3] M.M. Morcos, S. Zhang, K.D. Srivastava, and S.M. Gubanski "Dynamics of Metallic particle contamination in GIS with dielectric coating electrodes", IEEE Transactions on Power Delivery Vol. 15, No. 2, April 2000 p.p. 455-460.



- [4] J. Amarnath, B. P. Singh, C. Radhakrishna, S. Kamakshiah "Determination of particle trajectory in a Gas Insulated Busduct predicted by Monte-Carlo technique", CEIDP, 1999 during 17-21st October 1999, Texas, Austin, USA.
- [5] J. Amarnath et al., "Influence of Power Frequency and Switching Impulse Voltage on Particle Movement in GIS – Predicted by Monte-Carlo Technique" International IEEE, High Voltage Workshop, during April 18-21, 2001, California, USA.
- [6] J. Amarnath et al., "Influence of Power Frequency and Impulse Voltage on Motion of Metallic particle in a coated GIS – predicted by Monte-Carlo Technique" 6th International Seminar on Electrical and Electronics Insulating Materials and Systems (IEEMA) during November 23-24, 2000, Hyderabad, INDIA
- [7] J. Amarnath et al., "Movement of Metallic Particles in Gas Insulated Substations under the influence of various types of voltages" 11th National Power System Conference (NPSC-2000) during December 20-22, 2000, IISc, Bangalore, INDIA.
- [8] J. Amarnath et al., "Effect of Various Parameters of Movement of Metallic Particles in GIS" at International Seminar on Compact Substations and Gas Insulated Switch Gears (CBIP) during January 19-21, 2001, New Delhi, INDIA.
- [9] J. Amarnath et al., "Particle Trajectory in a Common Enclosure Three phase SF₆ Busduct" 12th International Symposium on High Voltage Engineering during August 20-24, 2001, IISc, Bangalore, INDIA.
- [10] G.V.Nagesh Kumar, J. Amarnath, B.P. Singh, K.D. Srivastava, "Electric Field Effect on Metallic Particle Contamination in a Common Enclosure Gas Insulated Busduct" IEEE Transactions on Dielectrics and Electrical Insulation Vol. 14, No. 2, 2007



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Effect of Multi FACTS Devices on the Enhancement of Available Transfer Capability in a Deregulated Power System

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Abstract

In a deregulated power system structure, power producers and customers share a common transmission network for wheeling power from the point of generation to the point of consumption. All parties in this open access environment may try to produce the energy from the cheaper source for greater profit margin, which may lead to overloading and congestion of certain corridors of the transmission network. This may result in violation of line flow, voltage and stability limits and there by undermine the system security, utilities therefore need to enhance “available transfer capability (ATC)” to ensure the system reliability is maintained while serving a wide range of bilateral and multilateral transactions. This paper focuses on the enhancement ATC using multi FACTS devices i.e combination of Thyristor Controlled Series Capacitor(TCSC) and Thyristor Controlled Phase Angle Regulator (TCPAR). The optimal location of FACTS devices were determined based on Sensitivity methods. The Reduction of Total System Reactive Power Losses Method was used to determine the suitable location of TCSC and TCPAR for ATC enhancement. The effectiveness of proposed method is demonstrated on modified IEEE-9 bus system and 24 bus real time system using power world simulator 8.0.¹

Keywords: Deregulation, Available Transfer Capability, TCSC, TCPAR, Multi FACTS devices.

1. Introduction

The introduction of deregulation has brought several new entities in the electricity market place, while on the other hand redefining the scope of activities of many of the existing players. With the ongoing expansion and growth of the electric utility industry, numerous changes are continuously being introduced.

Improved utilization of the existing power system is provided through the application of advanced control

technologies. Power electronics based equipment, or Flexible AC Transmission Systems (FACTS), provide proven technical solutions to address these new operating challenges being presented today. Thyristor Controlled Series Capacitor (TCSC), Thyristor Controlled Phase Angle Regulator (TCPAR), Unified Power Flow Controller (UPFC) and Static VAR Compensator (SVC) are some of the commonly used FACTS controllers. In many deregulated markets, the power transaction between buyer and seller is allowed based on calculation of ATC. Low ATC signifies that the network is unable to accommodate further transaction and hence does not promote free competition. FACTS controllers like TCSC, TCPAR can help to improve ATC by allowing more power transactions.

Electrical energy plays an important role in today's modern society. It is an efficient energy used widely for lighting, communication systems, transportation systems, industrial purposes and many other areas. In past, the electricity industry is in the control of government and therefore monopolistic. However over the past decade, the electric industry in many countries has undergone significant changes to reform into a free market, which is also known as deregulation.

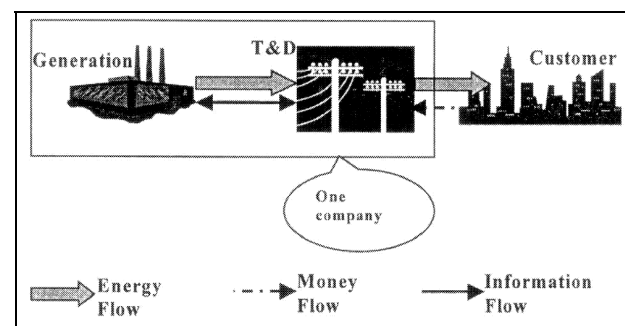


Figure 1. Typical structure of a vertically integrated utility

Figure 1. shows the typical structure of a vertically integrated utility where links of information flow existed only between the generators and the transmission system.

¹ This study has been implemented on Power World Simulator at NRI Institute of Technology, Agiripalli.



Similarly, money (cash) flow was unidirectional, from the consumer to the electric utility. One of the first step in the restructuring process of the power industry is the separation of the transmission activities from the electricity generation activities. The subsequent step was to introduce competition in generation activities.

An important point to note is that the restructuring process was started with the breaking up of a large vertically integrated utility which was characterized by the opening up of small municipal monopolies to competition.

A system operator for the whole system was appointed and it was entrusted with the responsibility of keeping the system in balance, *i.e.* to ensure that the production and imports continuously matched the consumption and exports. Quite appropriately, the system operator came to be known as the Independent System Operator (ISO).

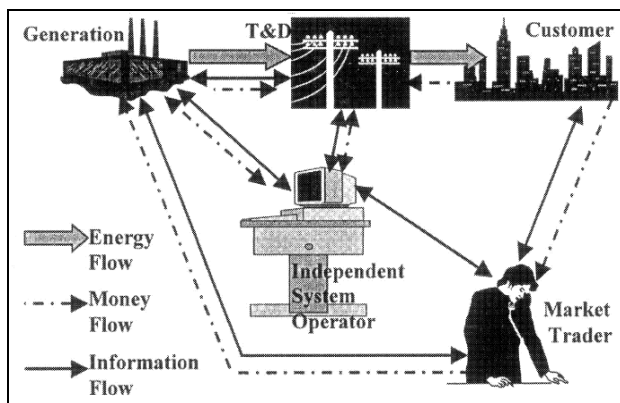


Figure 2. Typical structure of a deregulated electricity system

Figure 2 shows the typical structure of a deregulated electricity system with links of information and money (cash) flow between various players. The possibility of having such a complex nature of information flow has been one of the driving factors in the process of deregulation of the power sector. This has been possible due to the rapid developments in the fields of communication and information technology during the nineties decade.

2. FACTS CONTROLLERS

2.1 Thyristor Controlled Series Compensator

Thyristor-controlled series capacitor (TCSC) is a capacitive reactance compensator, which consists of a series capacitor bank shunted by a thyristor controlled reactor in order to provide a smoothly variable series capacitive reactance.

Even though a TCSC in the normal operating range is mainly capacitive, but it can also be used in an inductive mode. The power flow over a transmission line can be increased by controlled series compensation with minimum risk of sub synchronous resonance (SSR). TCSC is a second generation FACTS controller, which controls the impedance of the line in which it is connected by varying the firing angle of the thyristors. A TCSC module comprises a series fixed capacitor that is connected in parallel to a thyristor controlled reactor (TCR).

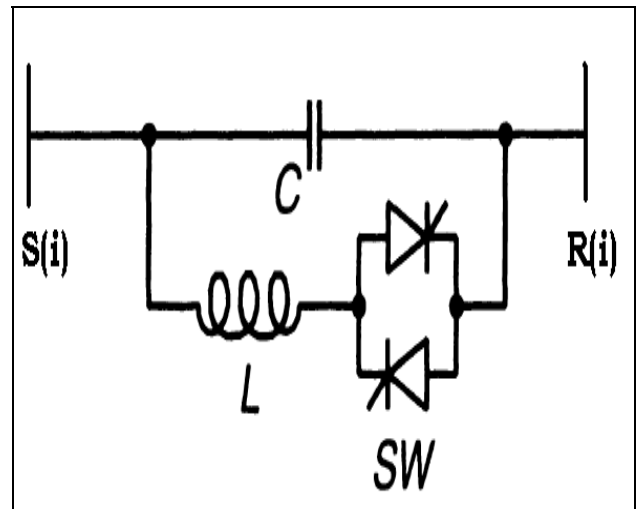


Figure 3. TCSC Module

A TCR includes a pair of anti-parallel thyristors that are connected in series with an inductor. In a TCSC, a metal oxide varistor (MOV) along with a bypass breaker is connected in parallel to the fixed capacitor for overvoltage protection. A complete compensation system may be made up of several of these modules.

2.2 Thyristor Controlled Phase Angle Regulator

TCPAR is identical with a phase shifting transformer with a thyristor type tap changer and plays an important role in increasing load ability of the existing system and controlling the congestion in the network. FACTS device like TCPAR can be used to regulate the power flow in the tie-lines of interconnected power system.

This is also known as Static Phase Shifter (SPS) and phase shift with respect to the bus voltage is achieved by adding or subtracting a variable voltage component in quadrature with the bus voltage. The quadrature voltage is injected in series with the transmission line by a boosting transformer. The basic arrangement is shown in figure 4.

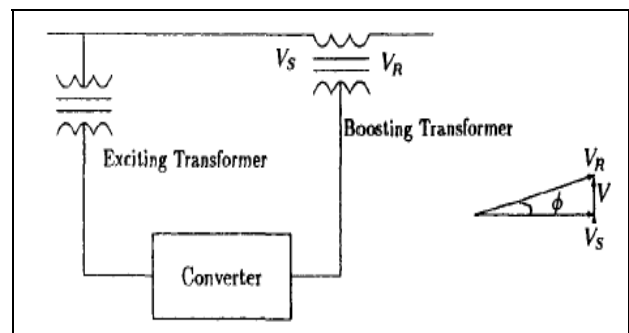


Figure 4. Shows Static Phase Shifter

3. MODELING OF FACTS DEVICES

For enhancing of transfer capability, the static models of these controllers are considered. It is assumed that the time constants in FACTS devices are very small and hence this approximation is justified.



3.1 Analysis of Transmission Lines and its Power Flows and Loss

Let the complex voltages at bus i and bus j be denoted as $V_i \angle \delta_i$ and $V_j \angle \delta_j$ respectively.

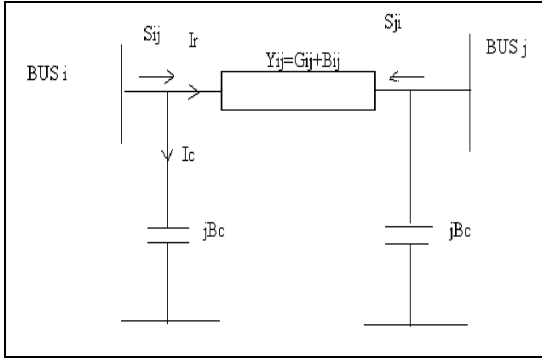


Figure 5. Model of a Transmission line.

The complex power flowing from bus i to bus j can be expressed as:

$$\begin{aligned} S_{ij}^* &= P_{ij} - jQ_{ij} = V_i^* I_{ij} = V_i^* (I_R + I_C) \\ &= V_i^* [(V_i - V_j)(G_{ij} + jB_{ij}) + V_i(jB_c)] \\ &= V_i^2 [G_{ij} + j(B_{ij} + B_c)] - V_i^* V_j (G_{ij} + jB_{ij}) \\ &= [V_i^2 G_{ij} - V_i V_j G_{ij} \cos(\delta_j - \delta_i) + V_i V_j B_{ij} \sin(\delta_j - \delta_i)] \\ &\quad + j[V_i^2 (B_{ij} + B_c) - V_i V_j B_{ij} \cos(\delta_j - \delta_i) - V_i V_j G_{ij} \sin(\delta_j - \delta_i)] \end{aligned} \quad (1)$$

The Active & Reactive power flow from bus i to bus j is,

$$P_{ij} = V_i^2 G_{ij} - V_i V_j G_{ij} \cos(\delta_{ij}) - V_i V_j B_{ij} \sin(\delta_{ij}) \quad (2)$$

$$\begin{aligned} Q_{ij} &= -V_i^2 (B_{ij} + B_c) + V_i V_j B_{ij} \cos(\delta_{ij}) \\ &\quad - V_i V_j G_{ij} \sin(\delta_{ij}) \end{aligned} \quad (3)$$

Where $\delta_{ij} = \delta_i - \delta_j$

Similarly, Real & Reactive Power flows from bus j to bus i can be,

$$P_{ji} = V_j^2 G_{ij} - V_i V_j G_{ij} \cos(\delta_{ij}) + V_i V_j B_{ij} \sin(\delta_{ij}) \quad (4)$$

$$Q_{ji} = -V_j^2 (B_{ij} + B_c) + V_i V_j B_{ij} \sin(\delta_{ij}) + V_i V_j G_{ij} \cos(\delta_{ij}) \quad (5)$$

The active and reactive Power loss in the line can be calculated as,

$$P_L = P_{ij} + P_{ji}$$

$$\begin{aligned} P_L &= V_i^2 G_{ij} - V_i V_j G_{ij} \cos(\delta_{ij}) - V_i V_j B_{ij} \sin(\delta_{ij}) \\ &\quad + V_j^2 G_{ij} - V_i V_j G_{ij} \cos(\delta_{ij}) + V_i V_j B_{ij} \sin(\delta_{ij}) \end{aligned}$$

$$P_L = V_i^2 G_{ij} + V_j^2 G_{ij} - 2V_i V_j G_{ij} \cos \delta_{ij} \quad (6)$$

$$Q_L = Q_{ij} + Q_{ji}$$

$$\begin{aligned} Q_L &= -V_i^2 (B_{ij} + B_c) + V_i V_j B_{ij} \cos(\delta_{ij}) - V_i V_j G_{ij} \sin(\delta_{ij}) \\ &\quad - V_j^2 (B_{ij} + B_c) + V_i V_j B_{ij} \sin(\delta_{ij}) + V_i V_j G_{ij} \cos(\delta_{ij}) \\ Q_L &= -V_i^2 (B_{ij} + B_c) - V_j^2 (B_{ij} + B_c) + 2V_i V_j B_{ij} \cos(\delta_{ij}) \end{aligned} \quad (7)$$

3.2 Power Injection Model Of Thyristor Controlled Series Compensator (TCSC)

Thyristor controlled series compensators (TCSC) are connected in series with the lines. The effect of a TCSC on the network is as a controllable reactance in the related transmission line which compensates the inductive reactance of the line results in reducing the transfer reactance between the buses. This leads to an increase in the maximum power that can be transferred on that line in addition to a reduction in the effective reactive power losses. The series capacitors also contribute to an improvement in the voltage profiles.

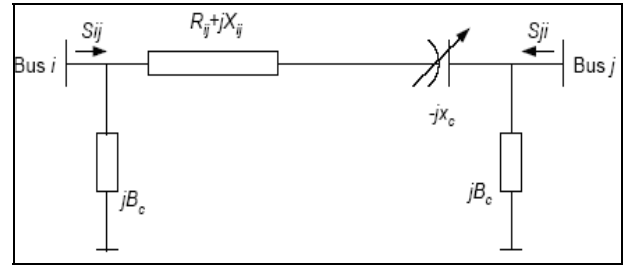


Figure 6. Model of a TCSC

Figure 6 shows a model of a transmission line with a TCSC connected between buses i and j . The transmission line is represented by its lumped π -equivalent parameters connected between the two buses. During the steady state, the TCSC can be considered as a static reactance $-jX_c$. This controllable reactance, X_c is directly used as the control variable to be implemented in the power flow equation.

Let the complex voltages at bus i and bus j be denoted as $V_i \angle \delta_i$ and $V_j \angle \delta_j$ respectively.

The expressions for real and reactive power flows from bus i to bus j can be written as from

$$P'_{ij} = V_i^2 G'_{ij} - V_i V_j [G'_{ij} \cos(\delta_{ij}) + B'_{ij} \sin(\delta_{ij})] \quad (8)$$

$$Q'_{ij} = -V_i^2 (B'_{ij} + B_c) - V_i V_j [G'_{ij} \sin(\delta_{ij}) + B'_{ij} \cos(\delta_{ij})] \quad (9)$$

Similarly, the real and reactive power flows from bus j to bus i can be expressed as,

$$P'_{ji} = V_j^2 G'_{ij} - V_i V_j [G'_{ij} \cos(\delta_{ij}) - B'_{ij} \sin(\delta_{ij})] \quad (10)$$



$$Q'_{ji} = -V_j^2(B'_{ij} + B_c) + V_i V_j [G'_{ij} \sin(\delta_{ij}) + B'_{ij} \cos(\delta_{ij})] \quad (11)$$

The active and reactive power losses in the line with TCSC are

$$P'_L = P'_{ij} + P'_{ji} = G'_{ij}(V_i^2 + V_j^2) - 2V_i V_j G'_{ij} \cos \delta_{ij} \quad (12)$$

$$\begin{aligned} Q'_L &= Q'_{ij} + Q'_{ji} \\ &= -(V_i^2 + V_j^2)(B'_{ij} + B_c) + 2V_i V_j G'_{ij} \cos \delta_{ij} \end{aligned} \quad (13)$$

Line flows

Without TCSC ($-jX_c$)

$$G_{ij} + jB_{ij} = \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} + j \frac{-(x_{ij})}{r_{ij}^2 + x_{ij}^2}$$

With TCSC ($-jX_c$)

$$G'_{ij} + jB'_{ij} = \frac{1}{r_{ij} + j(x_{ij} - x_c)} * \frac{r_{ij} - j(x_{ij} - x_c)}{r_{ij} - j(x_{ij} - x_c)}$$

where

$$G'_{ij} = \frac{r_{ij}}{r_{ij}^2 - j(x_{ij} - x_c)^2}$$

and

Hence, $\Delta G_{ij} = G_{ij} - G'_{ij}$ (without x_c - with x_c)

$$\begin{aligned} \Delta G_{ij} &= \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} - \frac{r_{ij}}{r_{ij}^2 - j(x_{ij} - x_c)^2} \\ &= r_{ij} * \frac{r_{ij}^2 + (x_{ij} - x_c)^2 - r_{ij}^2 - x_{ij}^2}{(r_{ij}^2 + x_{ij}^2)\{r_{ij}^2 - j(x_{ij} - x_c)^2\}} \\ \Delta G_{ij} &= \frac{x_c r_{ij}(x_c - 2x_{ij})}{(r_{ij}^2 + x_{ij}^2)\{r_{ij}^2 - j(x_{ij} - x_c)^2\}} \end{aligned} \quad (14)$$

$$\Delta B_{ij} = \frac{-x_c(r_{ij}^2 - x_{ij}^2 + x_c x_{ij})}{(r_{ij}^2 + x_{ij}^2)\{r_{ij}^2 - j(x_{ij} - x_c)^2\}} \quad (15)$$

Hence, the Change in line flows due to Series Capacitance,

Hence, the Change in line flows due to Series Capacitance,

The real Power injection at bus 'i'

$$P'_i = V_i^2 \Delta G_{ij} - V_i V_j [\Delta G_{ij} \cos(\delta_{ij}) + \Delta B_{ij} \sin(\delta_{ij})] \quad (16)$$

$$P'_j = V_j^2 \Delta G_{ij} - V_i V_j [\Delta G_{ij} \cos(\delta_{ij}) + \Delta B_{ij} \sin(\delta_{ij})] \quad (17)$$

$$Q'_i = -V_i^2 \Delta B_{ij} - V_i V_j [\Delta G_{ij} \sin(\delta_{ij}) - \Delta B_{ij} \cos(\delta_{ij})] \quad (18)$$

$$Q'_j = -V_j^2 \Delta B_{ij} + V_i V_j [\Delta G_{ij} \sin(\delta_{ij}) + \Delta B_{ij} \cos(\delta_{ij})] \quad (19)$$

These equations are used to structure the TCSC to Enhance the Power Transfer Capability.

3.3 Power Injection of Thyristor Controlled Phase Angle Regulator (TCPAR)

In a Thyristor controlled phase angle regulator, the phase shift is achieved by introducing a variable voltage component in perpendicular to the phase voltage of the line. The static model of a TCPAR having a complex tap ratio of $1:a \angle \alpha$ and a transmission line between bus i and bus j is Shown in Figure5

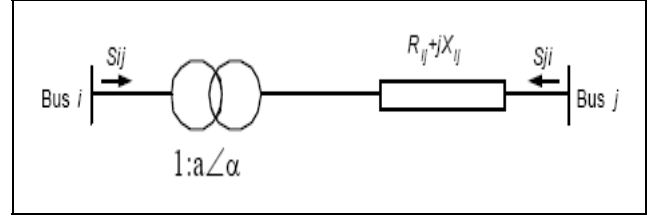


Figure 7. Model of TCPAR

The real and reactive power flows from bus i to bus j can be expressed as

$$\begin{aligned} S_{ij}^* &= P_{ij} - jQ_{ij} \\ &= V_i^* [(V_i - V_j)] Y_{ij} \end{aligned} \quad (20)$$

$$\begin{aligned} S_{ij}^* &= V_i^* [a^2 V_i - a^* V_j] Y_{ij} \\ &= V_i^* [a^2 V_i - a^* V_j] [G_{ij} + jB_{ij}] \\ &= V_i^* a^2 V_i (G_{ij} + jB_{ij}) - V_i^* V_j a^* (G_{ij} + jB_{ij}) \\ &= V_i^2 a^2 (G_{ij} + jB_{ij}) - V_i V_j a (\angle(\delta_j - \delta_i - \alpha)) \\ &= V_i^2 a^2 G_{ij} + jV_i^2 a^2 B_{ij} - V_i V_j a (G_{ij} + jB_{ij}) \cos(\delta_j - \delta_i - \alpha) \\ &\quad - jV_i V_j a (G_{ij} + jB_{ij}) \sin(\delta_j - \delta_i - \alpha) \\ &= V_i^2 a^2 G_{ij} + jV_i^2 a^2 B_{ij} - V_i V_j a G_{ij} \cos(\delta_j - \delta_i - \alpha) \\ &\quad - jV_i V_j a B_{ij} \cos(\delta_j - \delta_i - \alpha) \\ &\quad - jV_i V_j a G_{ij} \sin(\delta_j - \delta_i - \alpha) + jV_i V_j a B_{ij} \sin(\delta_j - \delta_i - \alpha) \\ &= [V_i^2 a^2 G_{ij} - V_i V_j a G_{ij} \cos[-(\delta_i - \delta_j + \alpha)] \\ &\quad + V_i V_j a B_{ij} \sin[-(\delta_i - \delta_j + \alpha)]] \\ &\quad + j[V_i^2 a^2 B_{ij} - V_i V_j a B_{ij} \cos[-(\delta_i - \delta_j + \alpha)] \\ &\quad - V_i V_j a G_{ij} \sin[-(\delta_i - \delta_j + \alpha)]] \end{aligned}$$



$$\begin{aligned}
&= [V_i^2 a^2 G_{ij} - V_i V_j a G_{ij} \cos(\delta_i - \delta_j + \alpha) \\
&- V_i V_j a B_{ij} \sin(\delta_i - \delta_j + \alpha)] \\
&+ j[V_i^2 a^2 B_{ij} - V_i V_j a B_{ij} \cos(\delta_i - \delta_j + \alpha) \\
&+ V_i V_j a G_{ij} \sin(\delta_i - \delta_j + \alpha)]
\end{aligned} \quad (21)$$

The real and reactive power flows from bus i to bus j are

Complex power, $S_{ij} = P_{ij} + jQ_{ij}$

$$\begin{aligned}
P_{ij} &= \text{Re}[S_{ij}^*] = \text{Re}\{V_i^* [a^2 V_i - a^* V_j] Y_{ij}\} \\
P_{ij} &= V_i^2 a^2 G_{ij} - V_i V_j a G_{ij} \cos(\delta_i - \delta_j + \alpha) \\
&- V_i V_j a B_{ij} \sin(\delta_i - \delta_j + \alpha)
\end{aligned} \quad (22)$$

$$\therefore Q_{ij} = -\text{Im}[S_{ij}^*] = \text{Im}\{V_i^* [a^2 V_i - a^* V_j] Y_{ij}\}$$

$$\begin{aligned}
Q_{ij} &= -[V_i^2 a^2 B_{ij} - V_i V_j a B_{ij} \cos(\delta_i - \delta_j + \alpha) \\
&+ V_i V_j a G_{ij} \sin(\delta_i - \delta_j + \alpha)] \\
Q_{ij} &= -V_i^2 a^2 B_{ij} + V_i V_j a B_{ij} \cos(\delta_i - \delta_j + \alpha) \\
&- V_i V_j a G_{ij} \sin(\delta_i - \delta_j + \alpha)
\end{aligned} \quad (23)$$

Similarly, the real and reactive power flows from bus j to bus i are

$$\begin{aligned}
P_{ji} &= \text{Re}\{V_j^* [V_j - a V_i] Y_{ij}\} \\
P_{ji} &= V_j^2 G_{ij} - V_i V_j a G_{ij} \cos(\delta_i - \delta_j + \alpha) \\
&+ V_i V_j a B_{ij} \sin(\delta_i - \delta_j + \alpha)
\end{aligned} \quad (24)$$

$$\begin{aligned}
\text{And } Q_{ji} &= -\text{Im}\{V_j^* [V_j - a V_i] Y_{ij}\} \\
Q_{ji} &= -V_j^2 B_{ij} + V_i V_j a B_{ij} \cos(\delta_i - \delta_j + \alpha) \\
&+ V_i V_j a G_{ij} \sin(\delta_i - \delta_j + \alpha)
\end{aligned} \quad (25)$$

The real and reactive power loss in the line having a TCPAR can be expressed as

$$\begin{aligned}
P_L &= P_{ij} + P_{ji} \\
P_L &= V_i^2 a^2 G_{ij} + V_j^2 G_{ij} - 2 V_i V_j a G_{ij} \cos(\delta_i - \delta_j + \alpha)
\end{aligned} \quad (26)$$

$$\begin{aligned}
Q_L &= Q_{ij} + Q_{ji} \\
Q_L &= -V_i^2 a^2 B_{ij} - V_j^2 B_{ij} + 2 V_i V_j a B_{ij} \cos(\delta_i - \delta_j + \alpha)
\end{aligned} \quad (27)$$

This mathematical model makes the Y-bus asymmetrical. In order to make the Y-bus symmetrical, the TCPAR can be simulated by augmenting the existing line with additional power injections at the two buses. The injected active and reactive powers at bus i (ΔP_i , ΔQ_i) and bus j (ΔP_j , ΔQ_j) are given as:

$$\Delta P_i = -a^2 V_i^2 G_{ij} - a V_i V_j [G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j)] \quad (28)$$

$$\Delta P_j = -a V_i V_j [G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j)] \quad (29)$$

$$\Delta Q_i = a^2 V_i^2 B_{ij} + a V_i V_j [G_{ij} \cos(\delta_i - \delta_j) - B_{ij} \sin(\delta_i - \delta_j)] \quad (30)$$

$$\Delta Q_j = -a V_i V_j [G_{ij} \cos(\delta_i - \delta_j) - B_{ij} \sin(\delta_i - \delta_j)] \quad (31)$$

These equations will be used to model the TCPAR in the Enhancement of transfer capability

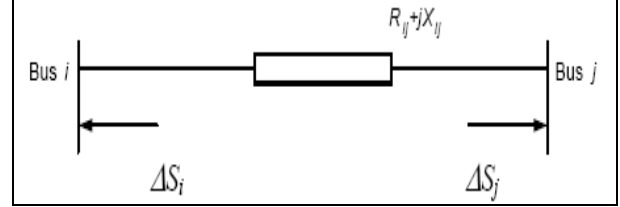


Figure 8. The injection model of the TCPAR

4. OPTIMAL LOCATION BASED ON SENSITIVITY APPROACH FOR TCSC AND TCPAR DEVICES

The static conditions are considered here for the placement of FACTS devices in the power system. The objectives for device placement may be one of the following:

1. Reduction in the real power loss of a particular line
2. Reduction in the total system real power loss
3. Reduction in the total system reactive power loss
4. Maximum relief of congestion in the system

4.1. Reduction of total system VAR power loss

Here, a method based on the sensitivity of the total system reactive power loss (Q_L) with respect to the control variables of the FACTS devices

For each of the two devices considered in Section 2.1 and 2.2, the following describes control parameters considered:

- Net line series reactance (X_{ij}) for a TCSC placed between buses i and j,
- Phase shift (α_{ij}) for a TCPAR placed between buses i and j.

The reactive power loss sensitivity factors with respect to these control variables may be given as follows:

1. Loss sensitivity with respect to control parameter X_{ij} of TCSC placed between buses i and j,

$$a_{ij} = \frac{\partial Q_L}{\partial X_{ij}} \quad (32)$$

2. Loss sensitivity with respect to control parameter θ_{ij} of TCPAR placed between buses i and j,

$$b_{ij} = \frac{\partial Q_L}{\partial \theta_{ij}} \quad (33)$$



These factors can be computed for a base case power flow solution. Consider a line connected between buses i and j and having a net series impedance of X_{ij} , that includes the reactance of a TCSC, if present. In that line θ_{ij} is the net phase shift in the line and includes the effect of the TCPAR. The loss sensitivities with respect to X_{ij} and θ_{ij} can be computed as:

$$a_{ij} = \frac{\partial Q_L}{\partial X_{ij}} = [V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j)] \frac{R_{ij}^2 - X_{ij}^2}{(R_{ij}^2 + X_{ij}^2)^2} \quad (34)$$

$$b_{ij} = \frac{\partial Q_L}{\partial \theta_{ij}} = [-2 a V_i V_j B_{ij} \sin(\theta_{ij})] \quad (35)$$

4.2. Selection of optimal placement of FACTS devices

Using the loss sensitivities as computed in the previous section, the criteria for deciding device location might be stated as follows:

1. TCSC must be placed in the line having the most positive loss sensitivity index a_{ij} .
2. TCPAR must be placed in the line having the highest absolute value of loss sensitivity index b_{ij} .

5. Results and Discussions:

Case studies are conducted on a 5 number of systems – IEEE 9, IEEE 24 rts .bus systems.

For each system, the enhancements of ATC and voltage profile at buses are determined with individual FACTS devices placed in turn and multi type FACTS devices. The following subsections describe the models and study conducted in detail.

The calculation of ATC is carried out by using Power world simulator to compute the power flow of each transfer case. The optimal power flow under normal and at contingency conditions has been carried out.

The static models of the FACTS controllers as given in Section 2.1, .2.2 are considered. TCSC is represented as a static reactance and TCPAR is considered as a transformer with a complex tap ratio. The optimal locations for placing each of these devices are determined by sensitivity analysis.

5.1. Case Study -1-IEEE 9 Bus system

5.1.1. System Model

This system consists of 9 buses, 9 line sections, 3 generator buses and 3 load buses, see figure 8.

5.1.2. Sensitivity Index

For this system, from Table1, three cases have been considered

- 1.A TCSC is placed in lines 4(4-5) and 7(6-7), operated with an inductive reactance of 75% and 20% of the line reactance;
- 2.A TCPAR is placed in the lines 3(3-6), 9(8-9) operated with a phase shift of 2.9, 4.5 degrees and unity tap ratio;
- 3.For Multi type FACTS (i.e. combination of TCSC & TCPAR), TCSC is placed in line 7(6-7) with 20% of the

line reactance and TCPAR is placed in line 3(3-6) with a phase shift of 2.9 degrees respectively.

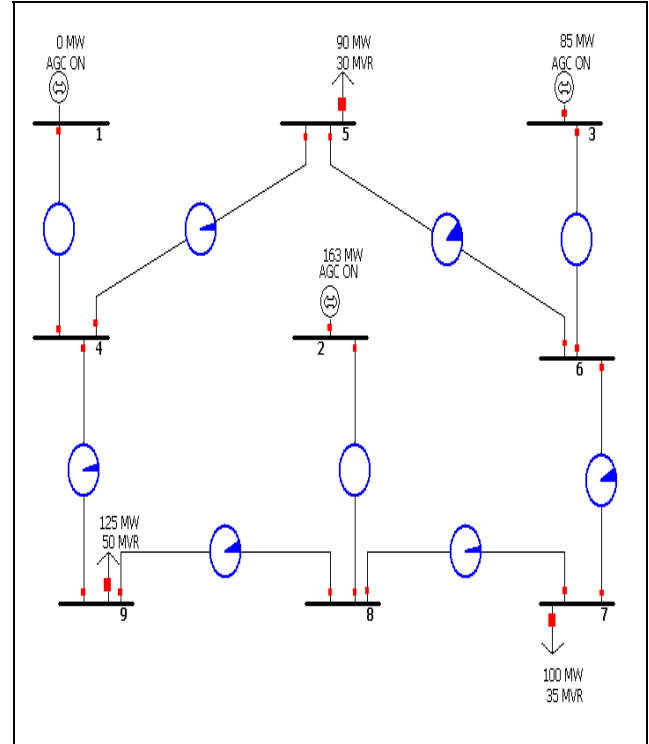


Figure 9. Single line diagram of IEEE 9 bus System

Line No(k)	From-To Bus	Loss Sensitivities	
		TCSC(a_{ij})	TCPAR (b_{ij})
1	1-4	-0.5771	1.441
2	8-2	-2.765	-3.26
3	3-6	-0.721	1.696
4	4-5	-0.095	0.588
5	9-4	-0.258	-0.752
6	5-6	-0.334	-1.19
7	6-7	-0.0785	0.4486
8	7-8	-0.58	-1.513
9	8-9	-0.711	1.679

TABLE 1. VAR loss sensitivity index of IEEE-9 Bus System



5.1.3. ATC Enhancement

From - To Bus	ATC (MW)						
	Without FACTS	With TCS C	% Enhancement	With TCP AR	% Enhancement	With TCS C - TCP AR	% Enhancement
1-4	170.4	170.4	N.E	170.9	0.251	171.4	0.583
2-8	124	124	N.E	124	N.E	124	N.E
3-5	112.9	121.6	7.103	113.0	0.044	122.9	8.13
3-8	135.6	135.9	0.228	142.6	4.920	145.6	6.865

Table 2. Available Transfer Capability for an IEEE 9-Bus System [without Contingency]

It is observed from the Table 2, the enhancement of ATC is of 8.13% with Multi type FACTS which is more than that due to placement of individual FACTS devices.

From - To Bus	ATC (MW)						
	Without FACTS	With TCS C	% Enhancement	With TCP AR	% Enhancement	With TCS C - TCP AR	% Enhancement
1-4	169.95	170.0	0.058	171.3	0.822	172.9	1.74
2-8	87	87	N.E	87	N.E	87	N.E
3-5	166.45	166.5	0.036	166.5	0.048	167.4	0.591
3-8	166.45	166.5	0.036	166.5	0.048	167.4	0.591

Table 3: Available Transfer Capability for an IEEE 9-Bus System [with Contingency]

It is observed from the Table 3, the percentage enhancement of ATC is of 1.740% with Multi type FACTS which is more than that due to placement of individual FACTS devices under contingency condition at line 8 (i.e line between bus 7 and bus 8 disconnected).

5.1.4. Voltage Profiles

It is also observed from the Fig 8 that with multi-type FACTS devices the voltage at bus 5 is increased from 0.96064p.u to 0.9618 p.u under contingency conditions compared to individual FACTS devices

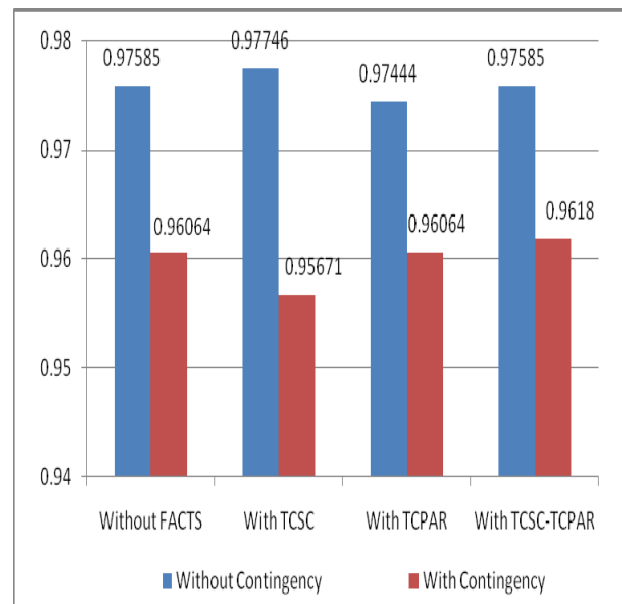


Figure 8. Comparison of voltages with and without FACTS under normal and contingency condition at bus 5

5.2. Case Study -2-IEEE 24 rts Bus system

5.2.1. System Model:

This system consists of 24 buses, 38 line sections, 11 generator buses and 17 load buses.

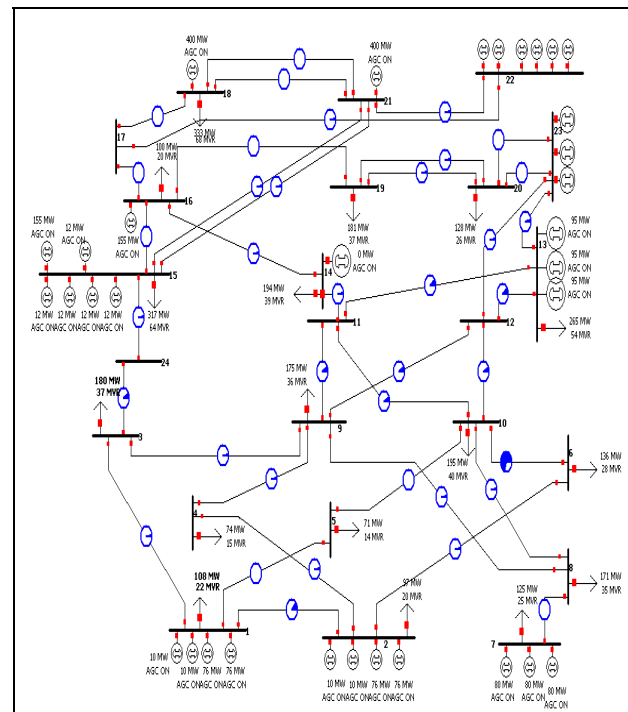


Figure 9. Single line diagram of IEEE 24 rts bus System



5.2.2. Sensitivity Index

Line No(k)	From-To Bus i-j	Loss sensitivities	
		TCSC(a _{ij})	TCPAR(b _{ij})
1	1-2	-0.01232	0.2421
2	1-3	-0.0474	-0.256
3	1-5	-0.2897	1.227
4	2-4	-0.154	0.6933
5	2-6	-0.3001	1.149
6	3-9	-0.0756	0.5534
7	3-24	-4.898	-4.33
8	4-9	-0.1096	-0.7256
9	5-10	-0.06604	-0.129
10	6-10	-1.0578	-2.146
11	7-8	-1.1189	2.218
12	8-9	-0.1122	-0.738
13	8-10	-0.116	-0.3062
14	9-11	-1.164	-2.1447
15	9-12	-0.01225	-2.44029
16	10-11	-2.922	-3.1667
17	10-12	-3.176	-3.4889
18	11-13	-0.877	-1.638
19	11-14	-3.322	-3.614
20	12-13	-0.368	-1.185
21	12-23	-4.986	-4.608
22	13-23	-4.72	-4.5339
23	14-16	-13.67	-7.308
24	15-16	-1.3055	2.351
25	15-21	-4.47	-4.2029
26	15-21	-4.47	-4.2029
27	15-24	-4.575	4.151
28	16-17	-9.865	-6.384
29	16-19	-1.409	2.421
30	17-18	-3.43	-3.594
31	17-22	-1.76	-2.825
32	18-21	-0.328	-1.233
33	18-21	-0.328	-1.233
34	19-20	-0.214	-0.571
35	19-20	-0.214	-0.571
36	20-23	-1.0022	-1.8325
37	20-23	-1.0022	-1.8325
38	21-22	-2.218	-3.201

Table 4. VAR loss sensitivity index of IEEE-24 rts Bus System

For this system, from Table 4, three cases have been considered (from Section 3.4.2)

1. A TCSC is placed in lines 1(1-2) and 15(9-12), operated with an inductive reactance of 75% and 20% of the line reactance;
2. A TCPAR is placed in the lines 23(14-16), 28(16-17) operated with a phase shift of 2.5, 4.5 degrees and unity tap ratio;
3. For Multi type FACTS (i.e. combination of TCSC & TCPAR), TCSC is placed in line 15(9-12) with 20% of the line reactance and TCPAR is placed in line 23(14-16) with a phase shift of 2.5 degrees respectively.

5.2.3. ATC Enhancement

From-To Bus	ATC (MW)						
	With out FACTS	With TCS C	% Enhancement	With TCP AR	% Enhancement	With TCS C- TCP AR	% Enhancement
1-5	155.0	155.8	0.50	157.9	1.81	158.3	2.046
23-15	817.2	821.2	0.49	817.2	N.E	827.2	1.208
13-21	917.9	938.6	2.20	927.6	1.04	947.6	3.135
16-21	1118.3	1123.8	0.49	1118.3	N.E	1183.5	5.511
22-19	332.4	332.4	N.E	332.4	N.E	337.4	1.493

Table 5. Available Transfer Capability for an IEEE 24 rts Bus System [without Contingency]

It is observed from the Table .5, the Enhancement of ATC is of 5.511% with Multi type FACTS which is more than that due to placement of individual FACTS devices.

It is observed from the Table 6, the percentage enhancement of ATC is of 5.66% with Multi type FACTS which is more than that due to placement of individual FACTS devices under contingency condition at line 10 (i.e line between bus 6 and bus 10 disconnected).



From - To Bus	ATC (MW)						
	Without FACTS	With TCSC	% Enhancement	With TCPAR	% Enhancement	With TCSC-TCPAR	% Enhancement
1-5	212.31	212.31	N.E	215.5	1.48	224.2	5.32
23-15	846.5	846.5	N.E	846.5	N.E	862.5	1.86
13-21	1007.7	1101.0	8.47	1037.5	2.87	1068.2	5.66
16-21	1159.7	1185.0	2.13	1159.7	N.E	1228.1	5.56
22-19	322.66	322.66	N.E	322.66	N.E	326.9	1.31

Table 6. Available Transfer Capability for an IEEE 24 rts Bus System [with Contingency at line10 (6-10)]

5.2.4. Voltage Profiles

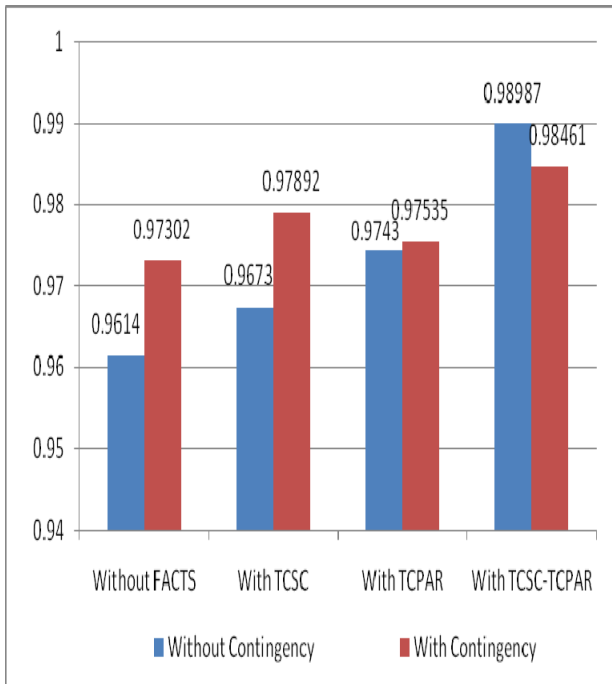


Figure 10. Comparison of voltages with and without FACTS under normal and contingency condition at bus 24

It is observed from the Fig10 that with multi-type FACTS devices the voltage at bus 24 is increased from 0.97302p.u to 0.98461 p.u under contingency conditions compared to individual FACTS devices.

6. Conclusions

It is observed that the enhancement of Available Transfer Capability comparison between various buses of an IEEE-9, 24rts, bus systems. Also observed that the enhancement of ATC using Multi type FACTS devices is more than individual FACTS devices like TCSC, TCPAR by placing optimally with sensitivity approach.

It is observed that the enhancement of ATC is not only for IEEE test systems but also for Indian utility system with and without FACTS devices under normal and contingency conditions.

Also observed the voltages with and without FACTS devices while enhancing the Available Transfer Capability. This indicates that the system is more voltage stable while enhancing ATC with multi type FACTS devices than with individual FACTS devices under contingency conditions.

Nomenclature

- P_{ij} Real power flow from bus i to j
 Q_{ij} Reactive power flow from i to j
 P_L Real power losses in the line.
 Q_L Reactive power losses in the line.
 P_i^1 Real power injection at bus i
 Q_i^1 Reactive power injection at bus i
 P_{ij}^1 Real power flow from bus i to j with TCSC
 Q_{ij}^1 Reactive power flow from bus i to j with TCSC
 P_L^1 Real power losses in the line with TCSC
 Q_L^1 Reactive power losses in the line with TCSC
 $V_i \angle \delta_i$ Complex voltage at bus i
 $V_j \angle \delta$ Complex voltage at bus j

REFERENCES

- [1] North American Electric Reliability Council, "Available Transfer Capability Definitions and Determination", NERC June 1996.
- [2] Hingorani, N.G, Gyugyi, L, "Understanding FACTS: Concepts and Technology of Flexible AC Transmission Systems", Institute of Electrical and Electronic Engineers Press, New York, 2000.
- [3] Ying Xiao, Y. H. Song, Chen-Ching Liu, and Y. Z. Sun, "Available Transfer Capability Enhancement Using FACTS Devices", IEEE Transactions On Power Systems, Vol. 18, No. 1, February 2003
- [4] Bairavan Veerayan Manikandan, Sathiasamuel Charles Raja, Paramasivam Venkatesh, "Enhancement of Available Transfer Capability with Facts Device in the Competitive Power Market", Engineering, Scientific Research, 2, 337-343, 2010.
- [5] J. Vara Prasad, I. Sai Ram, B. Jayababu, "Genetically Optimized FACTS Controllers for Available Transfer Capability Enhancement", International Journal of Computer Applications (0975 – 8887) Volume 19– No.4, April 2011
- [6] K. Narasimha Rao, J. Amarnath And K. Arun Kumar, "Voltage Constrained Available Transfer Capability Enhancement With Facts Devices",



ARPN Journal Of Engineering And Applied Sciences, Vol. 2, No. 6, December 2007

- [7] Ch.Rambabu, Dr.Y.P.Obulesu, Dr.Ch.Saibabu, "Improvement of Voltage Profile and Reduce Power System Losses by using Multi Type Facts Devices", International Journal of Computer Applications (0975 – 8887)Volume 13– No.2, January 2011
- [8] Claudio A. Canizares ,Alberto Berizzi ,Paolo Marannino, "Using FACTS Controllers to Maximize Available Transfer Capability", Bulk Power Systems Dynamics and Control IV{Restructuring, August 24-28, 1998, Santorini, Greece
- [9] H O Bansal, H P Agrawal, S Tiwana, A R Singal And L Shrivastava, "Optimal Location of FACT Devices to Control Reactive Power", International Journal of Engineering Science and Technology Vol. 2(6), 2010, 1556-1560
- [10]Yajing Gao, Ming Zhou, Jin Yang, "Available Transfer Capability Calculation Based on Contingency Selection", 5th IEEE Conference on Industrial Electronics and Applications, 2010.
- [11]G. MadhusudhanaRao, P. Vijaya Ramarao, T. Jayanth kumar, "Optimal Location of TCSC and SVC for Enhancement of ATC in a De-Regulated Environment using RGA", IEEE Transactions on Power Systems,2010.

Biographies



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An Efficient Novel Reduced Order Observer for State Estimation of an Internal Combustion Engine Model

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Abstract

In this paper, a reduced order observer is designed and implemented for state estimation of an internal combustion engine (ICE) model. In most engineering systems, certain states are directly measurable. Therefore, it is not cost effective and increases computational complexity to use a full order observer which will demand faster, more powerful processors. This is particularly the case in powertrain systems such as internal combustion engine. Here, we validate the use of a reduced order observer as opposed to a full order observer as a suitable solution to this problem. This is done through comparison, simulation and analysis of the results.

Keywords: *Reduced order observer, Internal combustion engine, Linear systems modelling, State estimation.*

1. Introduction

The introduction of a reduced dimension observer for the case where some of the state information is readily available was first introduced by Luenberger in [1]. The impact of the reduced order observer in state estimation and control has been greatly valuable ever since. Recently, various control strategies and schemes have been designed and implemented using a reduced order observer for state and/or parameter estimation [2-7]. Such systems include microalternators, induction motors, servo motors, reluctance motors, automatic transmissions and permanent magnet synchronous machines [8-12]. Previous work involving the design and implementation of a reduced observer can be found in this journal's past publications [13-14].

In all of the aforementioned systems, the reduced order observer was used on linear systems with directly measurable states to estimate states that are immeasurable otherwise. The advantage of reducing the observer's dimension is obvious; a lower order observer is analytically less complex. In practice, this means fewer sensors, which reduce the cost of the overall system. Reduced order ICE models can be found throughout literature [15-17]. Similarly to the work

presented here, these models utilize techniques to reduce, and therefore, simplify a higher order system. While each of the papers referenced present good simulation and implemented results compared to actual data, in our work we are concerned with validating that the reduced order observer performs better or just as well as the full order observer. Moreover, for ICEs specifically, reducing the size of the observer without reducing the size of the model beforehand has never been investigated. There has been much work in recent years to model the dynamics of the internal combustion engine (ICE) [18-23]. The ICE has become so complex that the engine is usually split into subsystems, each with its different tasks and constraints. A critical aspect of these tasks related to state estimation can provide the best fuel economy and emissions efficiency. Another important task is finding ways to reduce manufacturing cost by reducing the amount of redundant components. Such components include sensors, which can be unreliable, expensive and sensitive to environmental changes (pressure, temperature, water etc...). Indeed, electronic control units (ECUs) used in modern powertrain systems in today's vehicles already depend on well-designed observers to estimate states to work with state feedback control.

While the reduced order observer's ability to estimate and to control is well documented, it has not been utilized in engine models. In this paper, a full order observer is designed to estimate all the states of a third order ICE model. A comparison is then made between the performances of the full order observer to that of reduced observers. A simple step input command is used for all observer simulations and we analyse the steady state behaviour of the responses. Through comparison of these observers' performance, we validate that a reduced order observer can perform better or just as well as a full order observer.

In section 2, an outline of the reduced observer is given. Section 3 provides the design and implementation of the reduced observer on the ICE model. In section 4, an analysis of the results is discussed and corresponding simulations are provided. Section 5 is a conclusion of the work done in this paper.



2. Reduced Order Observer Theory

The design of a full order observer is well known and can be found in [1]. Here we outline the design for the reduced order observer used.

For a Linear Time Invariant (LTI) system written as:

$$\dot{x} = Ax(t) + Bu(t) \quad (1)$$

$$y = Cx(t) \quad (2)$$

where A, B and C are the state, input and output constant matrices, respectively. A, B and C have dimension $n \times n$, $r \times n$ and $m \times n$, respectively. n is the number of states in the system, r is the number of inputs and m is the number of directly measurable states used as outputs. A transformation matrix, T, is chosen,

$$T = \begin{bmatrix} C \\ D \end{bmatrix} \quad (3)$$

where D is a matrix that needs to be chosen to make the transformation matrix the appropriate dimensions and non-singular. Using prior knowledge of the structure of the output matrix, C, matrix D is chosen and the transformation matrix can be written as:

$$T = \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} I_m & 0_{n-m} \\ 0_m & I_{n-m} \end{bmatrix} \quad (4)$$

with I_m and I_{n-m} being identity matrices indexed by their respective dimensions. Similarly, 0_{n-m} and 0_m are zero matrices indexed by their respective dimensions. Matrix D is chosen to be the $(n-m) \times n$ matrix $\begin{bmatrix} 0_m & I_{n-m} \end{bmatrix}$, which ensures that the transformation matrix is always non-singular, as long as the output matrix, C, is the $m \times n$ matrix $\begin{bmatrix} I_m & 0_{n-m} \end{bmatrix}$. A new state vector is then derived using the transformation matrix,

$$w = Tx \quad (5)$$

Substituting (3), the new state vector can be written as a relationship between the system output, y , containing m measurable states and a new matrix with $(n-m)$ states,

$$w = Tx = \begin{bmatrix} C \\ D \end{bmatrix} x = \begin{bmatrix} y \\ v \end{bmatrix} \quad (6)$$

v is a $(n-m) \times n$ matrix containing the states which need to be estimated. Now that the new state vector is derived, a new transformed linear time invariant state equation is derived. By manipulating (5), (7-8) are

derived.

$$\dot{w} = T\dot{x} \quad (7)$$

$$x = T^{-1}w \quad (8)$$

$$T\dot{x} = TAx + TBu \quad (9)$$

$$\dot{w} = TAT^{-1}w + TBu \quad (10)$$

The LTI system in (1-2) is multiplied by the transformation matrix, T, to get (9). Manipulation of (7-9) creates the new transformed LTI state equation:

$$\dot{w} = \hat{A}w + \hat{B}u \quad (11)$$

where $\hat{A}$ and $\hat{B}$ are the new state and input matrices, respectively. The new state equation can be written as follows,

$$\dot{w} = \begin{bmatrix} \dot{y} \\ \dot{v} \end{bmatrix} = \hat{A} \begin{bmatrix} y \\ v \end{bmatrix} + \hat{B}u \quad (12)$$

The vector v contains the unknown states, which need to be estimated. It is advantageous at this point to partition, $\hat{A}$ and $\hat{B}$ of the transformed state vector as shown here:

$$\dot{w} = \begin{bmatrix} \dot{y} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix} + \begin{bmatrix} \hat{B}_1 \\ \hat{B}_2 \end{bmatrix} u \quad (13)$$

$\hat{A}_{11}$, $\hat{A}_{12}$, $\hat{A}_{21}$ and $\hat{A}_{22}$ are matrices with dimensions $m \times m$, $m \times (n-m)$, $(n-m) \times m$ and $(n-m) \times (n-m)$, respectively. $\hat{B}_1$ and $\hat{B}_2$ have dimensions $m \times r$ and $(n-m) \times r$, respectively. The expansion of (13) gives two separate equations:

$$\dot{y} = \hat{A}_{11}y + \hat{A}_{12}v + \hat{B}_1u \quad (14)$$

$$\dot{v} = \hat{A}_{21}y + \hat{A}_{22}v + \hat{B}_2u \quad (15)$$

Vector v contains the remaining state variables, which need to be estimated with the use of an observer. This observer has reduced dimensions because the measurable states from the state vector are not included. The observer will have the form:

$$\dot{v}^* = (\hat{A}_{22} - G\hat{A}_{12})v^* + G(\dot{y} - \hat{A}_{11}y - \hat{B}_1u) + \hat{A}_{21}y + \hat{B}_2u \quad (16)$$

The following relationship, $z = v^* - Gy$, is used to get rid of the $\dot{y}$ term in (16). G is the observer gain of dimensions $(n-m) \times m$. The gain can be found using



gain determination methods such, as a Linear Quadratic Regulator (LQR), which calculates a gain that will minimize an error criterion as discussed in [24-25]. Another method to determine the gain is pole placement, which places the observer poles in the stable region of the s-plane to ensure the observer estimation converges to the state [26-27]. For this paper, we used the pole placement method. Even though the LQR method is more optimal since it minimizes an error criterion, the pole placement method is simpler in practice. It can be used to achieve control by simple trial and error and using certain rule of thumbs. However, the pole placement method has its limitations and its lack of ability to optimize the gain can lead to low observer performance. $\hat{V}^*$ and V^* indicate the estimated state variable vector of v . Getting rid of the differentiator term $\dot{y}$ gives the final observer form:

$$\dot{z} = (\hat{A}_{22} - G\hat{A}_{12})z + (\hat{A}_{22} - G\hat{A}_{12})G y + (\hat{A}_{21} - G\hat{A}_{11})y + (\hat{B}_2 - G\hat{B}_1)u \quad (17)$$

The reduced order observer model in (17) is now obtained as a linear relationship between the measurable states, y , the measurable input, u , and the estimated state vector, z , which contains the remainder of the system's states.

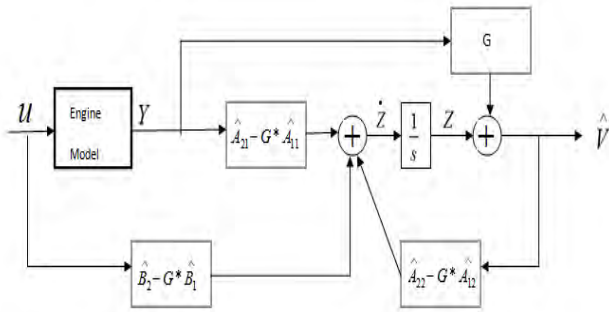


Figure 1. Reduced Order Observer Block Diagram

Figure 1 depicts a block diagram of the reduced order observer.

3. Design and implementation

The third order ICE model along with its parameters were created and thoroughly outlined by Baumgarther and Geering in [28]. The model, of a Chrysler Neon, was slightly adjusted for this paper. The inputs, u , were reduced from three to one. The state vector is as follows:

$$\begin{cases} x_1: \text{Throttle Angle (Degrees)} \\ x_2: \text{Intake Manifold Pressure (Torr)} \\ x_3: \text{Engine Speed (RPM)} \\ u: \text{Commanded Throttle Angle (Degrees)} \end{cases} \quad (18)$$

The mathematical model for nominal parameters is presented as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -35 & 0 & 0 \\ 4.212 & -3.822 & -6.552 \\ 0 & 3.2 & 1.84 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 35 \\ 0 \\ 0 \end{bmatrix} u \quad (19)$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

In order to present the fundamental stability of the model, we calculate the controllability and observability of the system. It is known in literature that by taking the absolute value of the determinant of the respective matrices, we are able to get a measure of just how controllable and observable the system is. The results are presented in Table 1. The absolute values of both determinants, presented in columns 2 and 4 of Table 1, yield two large numbers that are far from 0. This represents a system that is highly controllable and observable.

Observability Matrix (N)	det[N]
$N = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 3.2 & 1.84 \\ 13.478 & -6.34 & -17.581 \end{bmatrix}$	-43.131
Observability Matrix (M)	det[M]
$M = \begin{bmatrix} 35 & -122.5 & 42.87 \\ 0 & 147 & -57.23 \\ 0 & 0 & 472 \end{bmatrix}$	243

Table 1. Observability and controllability of ICE model

Full Order Observer design: First, a full order observer is designed to estimate all the states of the system. A standard Luenberger observer is designed using the form:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + G_1[y(t) - \hat{y}(t)] \quad (20)$$

$$\hat{y}(t) = C\hat{x}(t) \quad (21)$$

The parameters for (20-21) are presented in Table 2. $\hat{x}$ denotes the estimated state vector. The full observer gain which we shall refer to as G_1 was found using the pole placement method and placing the full observer poles ($A - G_1C$) at a location that would make them faster than the system's poles. To ensure the observer stabilizes well before the system, we made the observer poles three times faster. The corresponding observer gain and system's poles are also presented in Table 2.

Reduced Order Observer design (2nd Order): Next, we look at the case where the throttle command can directly be measured so there is no need to estimate it using an observer. The reduced observer, then, becomes a second order as opposed third order as the full observer. The transformation matrix (4) was chosen along with the partitioned state and input matrices. Next, we find the observer gain, G_2 , using the pole placement method. Similarly to the full order observer, the poles of the second order reduced observer (17), which



are $(\hat{A}_{22} - G_2 \hat{A}_{12})$, were placed at a location that makes them three times faster than the system's poles. All of the aforementioned parameters are presented in Table 3.

State Observer model system parameters	Parameter matrix
State Matrix	$A = \begin{bmatrix} -35 & 0 & 0 \\ 4.212 & -3.822 & -6.552 \\ 0 & 3.2 & 1.84 \end{bmatrix}$
Input Matrix	$B = \begin{bmatrix} 35 \\ 0 \\ 0 \end{bmatrix}$
Output Matrix	$C = [1 \ 1 \ 1]$
Gain Matrix	$G_1 = \begin{bmatrix} 2.1133 \\ 3.5262 \\ 26.7216 \end{bmatrix}$
System Poles	$\underline{\lambda} = \begin{bmatrix} -35 \\ -0.991 \pm 3.5981j \end{bmatrix}$

Table 2. Full observer design parameters

Reduced Observer model system parameters	Matrices
Partitioned State Matrices	$\hat{A}_{11} = [-35]$ $\hat{A}_{12} = [0 \ 0]$ $\hat{A}_{21} = \begin{bmatrix} 4.21 \\ 0 \end{bmatrix}$ $\hat{A}_{22} = \begin{bmatrix} -3.822 & -6.552 \\ 3.2 & 1.84 \end{bmatrix}$
Partitioned Input Matrices	$\hat{B}_1 = [35]$ $\hat{B}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
Gain Matrix	$G_2 = \begin{bmatrix} 1.3918 \\ 2.5662 \end{bmatrix}$
Transformation Matrix	$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Table 3. 2nd Order observer design parameters

Reduced Order Observer design (First Order): Lastly, we look at the case where both the throttle command and the manifold air pressure are both directly measurable. There is now only a need to estimate the engine speed. The reduced observer is now in its simplest form as a first order observer. The transformation matrix is the same as for the 2nd order reduced observer. The new partitioned state and input matrices have different dimensions as the 2nd order reduced observer. The observer gain, G_3 , is again found using the pole placement method. The poles of $(\hat{A}_{22} - G_3 \hat{A}_{12})$ were placed at a location that makes them three times faster than the system's poles. All of the parameters are presented in Table 4.

Reduced Observer model system parameters	Matrices
Partitioned State Matrices	$\hat{A}_{11} = \begin{bmatrix} -35 & 0 \\ 4.21 & -3.822 \end{bmatrix}$ $\hat{A}_{12} = \begin{bmatrix} 0 \\ -6.552 \end{bmatrix}$ $\hat{A}_{21} = [35 \ 3.2]$ $\hat{A}_{22} = [1.84]$
Partitioned Input Matrices	$\hat{B}_1 = \begin{bmatrix} 35 \\ 0 \end{bmatrix}$ $\hat{B}_2 = [0]$
Gain Matrix	$G_3 = [0.1623 \ -1.6953]$
Transformation Matrix	$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Table 4. 1st Order observer design parameters



4. Results and Analysis

In this section, the simulation results are presented and analysed. First, we take a look at the simulations of all 3 observers; Full order, 2nd order reduced and 1st order reduced. Figure 2 depicts all three states, which were estimated using the full order observer along with the actual measured values of the system. Figure 3 shows the result of the second order reduced observer states; manifold pressure and engine speed, and compares it to their corresponding full observer estimate and actual measurements from the systems. Figure 4 presents the simulation results of the first order reduced observer where the only state is the engine speed. The full observer and second observer estimate of the engine speed are also included for comparison, along with the actual measured speed from the system.

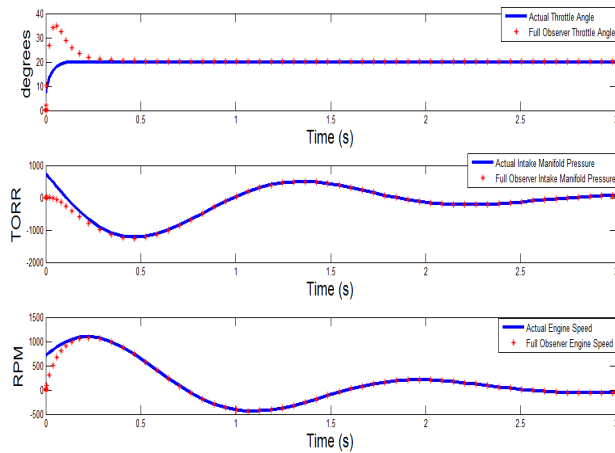


Figure 2. Simulation results of full observer

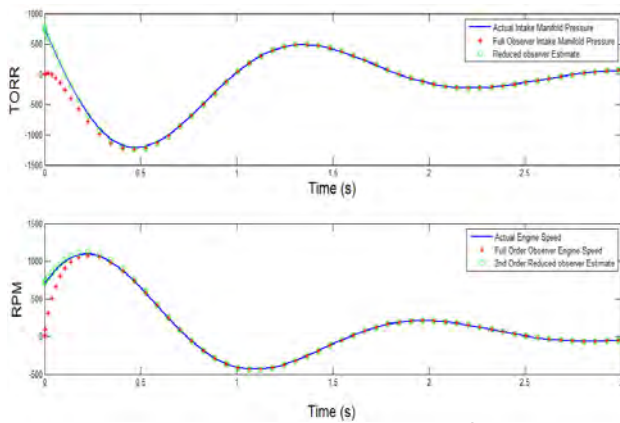


Figure 3. Simulation results of full observer and 2nd order reduced observer

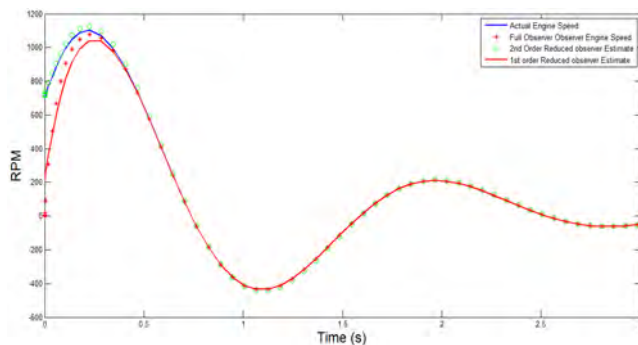


Figure 4. Simulation results of full observer, 2nd order reduced observer and 1st order reduced observer

Figures 5-7 depict the error dynamics of each of the observers: full, 2nd order reduced observer and 1st order reduced observer.

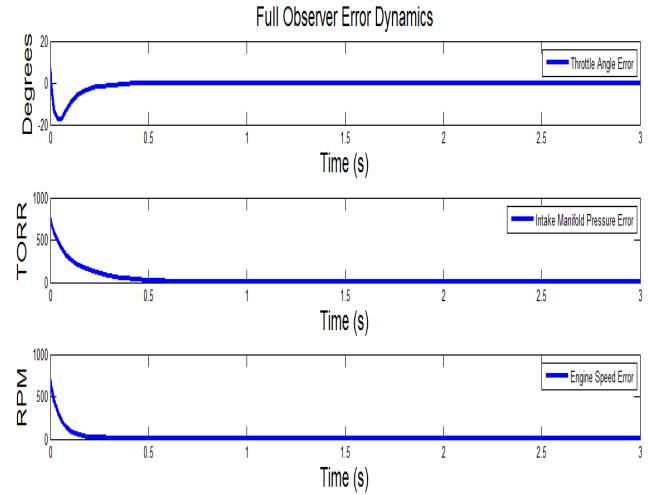


Figure 5. Error dynamics of full observer: Commanded throttle position, Intake manifold pressure and Engine speed

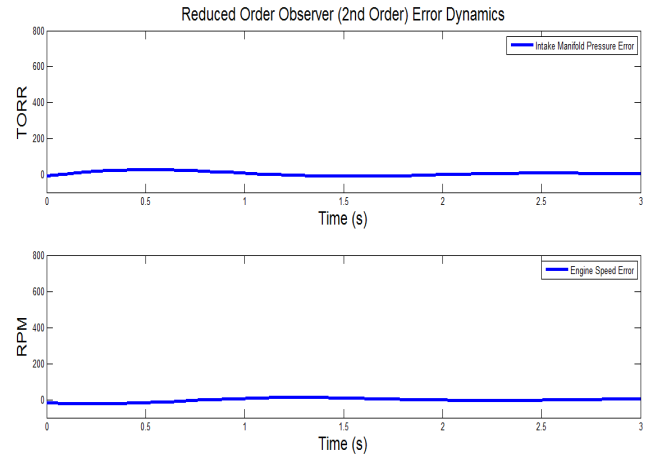


Figure 6. Error dynamics of 2nd order reduced observer: Intake manifold pressure and Engine speed

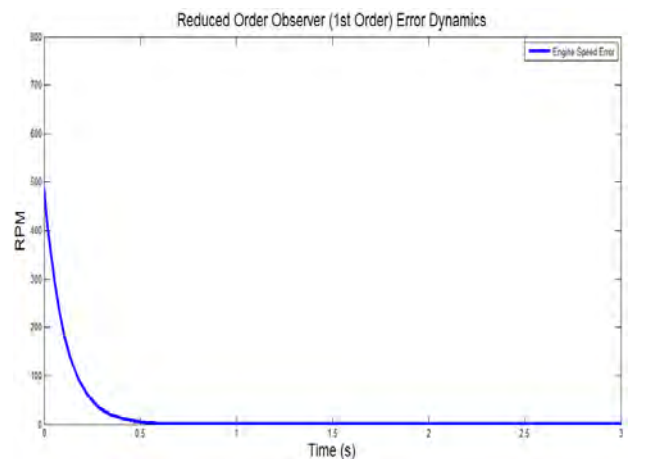


Figure 7. Error dynamics of 1st order reduced observer: Engine Speed

We take a special interest at the error dynamics to evaluate the performance of the reduced observers. All observers have been very well designed and visually their error converges towards zero asymptotically and quickly. Comparing the error dynamics for the manifold pressure



in the full and 2nd order reduced observer, it is evident that the reduced observer converges faster than its counterpart. It does seem, however, that the 2nd order reduced observer's average error throughout the entire simulation is higher than that of the full observer. In the same fashion, we take a look at the error dynamics of all three observers for the engine speed estimation. Again, in all three observers, the estimation converges quickly with the 2nd order reduced observer having the least initial error, and the full observer converged fastest. The 1st order reduced observer, nonetheless, was only about 250 ms slower to converge which is an insignificant amount of time.

5. Conclusion

In this paper, we addressed an issue related to state estimation of typical internal combustion engines. This issue involves the redundancy, high costs and unreliability that sensors are subject to and can cause. We validated the use of reduced observers as a solution. A reduced observer can estimate the unmeasurable states using measurements of those which are measured directly. This reduces the order of the observer, which otherwise would need to estimate more states. In practice, this allows for the removal of sensors. A third order ICE model was used for which we designed and implemented a full order observer and two reduced order observers to estimate its three states. The two reduced order observers were of second and first order, for the cases where 1 and then 2 states are directly measurable. An analysis of the error dynamics of all three observers was discussed. The reduced observers converged just as fast as or even faster than the full observer. Also, the reduced observer showed an insignificant amount of steady state error which was only slightly larger than or just as low as the full order observer. This demonstrates that the use of reduced order observers can be extended further to present day powertrain control modules, which already use observers in their algorithm. They can be utilized to cheapen cost, reduce unreliability and decrease system complexity.

6. References

- [1] D. Luenberger, "An Introduction to Observers," IEEE Transactions on Automatic Control, Vol. 16, No. 6, pp. 596-602, 1971.
- [2] S. Dashkovskiy and L. Naujok, "Quasi-ISS/ISDS reduced-order observers and quantized output feedback for interconnected systems," IEEE Conference in Decision and Control (CDC), 49th, pp. 5732-5737, 2010.
- [3] Y. Hamai, A. Inoue, Mingcong Deng and Y. Hirashima, "Duality between reduced-order observer and sliding mode controller and its application," in SICE 2004 Annual Conference, Vol. 3, pp. 2606-2609, 2004.
- [4] Liu Xiaofei, Jiao Xiaohong and Wang Chunyan, "Reduced-order observer-based robust control for cold rolling mill HAGC system with measurement delay," 29th Chinese Control Conference (CCC), pp. 2047-2052, 2010.
- [5] M. Mutsaers, S. Weiland and R. Engelaar, "Reduced-order observer design using a lagrangian method," 48th IEEE Conference in Decision and Control, pp. 5384-5389, 2009.
- [6] J. Solsona, M. I. Valla and C. Muravchik, "A nonlinear reduced order observer for permanent magnet synchronous motors," Industrial Electronics, IEEE Transactions on, vol. 43, pp. 492-497, 1996.
- [7] V.M. Raina, V.H. Quintana, M.A. Zohdy and J.H. Anderson, "Excitation System Stabiliser Design Through Minimisation Techniques", IEEE PES Summer meeting, 1975.
- [8] E.H. Okongwu, W.J. Wilson and J.H. Anderson, "Optimal State Feedback Control of a Microalternator using An Observer," IEEE Trans. On Power Apparatus and Systems, Iss. 2, pp. 594-602, 1978.
- [9] R. Aloui and N.B. Braiek, 'On the Determination of an Optimal State Observer Gain for Multivariable Systems: Application to Induction Motors,' Journal of Automation and Systems Engineering, 2008.
- [10] S. Yan, D. Xu, G. Wang, M. Yang, Y. Yu and X. Gui, "Low speed control of PMAC servo system based on reduced-order observer," IEEE International Conference on Intelligent Robots and Systems, pp. 4886-4889, 2006.
- [11] J. Solsona, M. Etchechoury, M. I. Valla and C. Muravchik, "A nonlinear reduced order observer for switched reluctance motors," Proceedings of the 32nd IEEE Conference on Decision and Control, pp. 3416-3417, vol.4., 1993.
- [12] J. Solsona, M. I. Valla and C. Muravchik, "A nonlinear reduced order observer for permanent magnet synchronous motors," IEEE Transactions on Industrial Electronics, vol. 43, pp. 492-497, 1996.
- [13] B. Sfaihi and O. Boubaker, "Piecewise Robust Observer for a Nonlinear Biological Process with unknown Inputs" International Journal of Artificial Intelligence and Machine Learning (AIML), vol. 6, Iss. 1, 2006.
- [14] R. Martinez-Guerra and R. Aguilar, "Risk Population for HIV Transmission using a Reduced Order Uncertainty and a Luenberger Observer" vol. 5, Iss. 2, 2005.
- [15] E. Kamei, H. Namba, K. Osaki and M. Ohba, "Application of reduced order model to automotive engine control system," in American Control Conference, 1987, 1987, pp. 1815-1820.
- [16] M. M. Kulkarni, R. P. Sinha and H. C. Dhariwal, "A reduced order model of turbo charged diesel engine," in American Control Conference, 1992, 1992, pp. 927-931.
- [17] C. G. Mayhew, K. L. Knierim, N. A. Chaturvedi, Sungbae Park, J. Ahmed and A. Kojic, "Reduced-order modeling for studying and controlling misfire in four-stroke HCCI engines," in Decision and Control, 2009 Held Jointly with the 2009 28th Chinese Control



- Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on, 2009, pp. 5194-5199.
- [18] J. Fiaschetti and N. Narasimhamurthi, "A Descriptive Bibliography of SI Engine modeling and control," SAE Technical paper, 1995.
- [19] J. A. Cook and B. K. Powell, "Modeling of an internal combustion engine for control analysis," IEEE Control Systems Magazine, vol. 8, pp. 20-26, 1988.
- [20] W. Wu and M. Ross, "Spark-Ignition Engine Fuel Consumptions Modeling" SAE Technical paper, 1999.
- [21] J.B. Haywood, "Internal Combustion Engines Fundamentals", McGraw-Hill, Inc., 1988.
- [22] C.F. Taylor, "Internal Combustion Engine in Theory and Practice", Vol 1 and 2, 2nd Edition, The MIT Press, Cambridge, MA 1988.
- [23] E. Hendricks, A. Chevalier, M. Jensen and S. Sorenson, "Modelling of the Intake Manifold Filling Dynamics", SAE International Congress and Exposition, 1996.
- [24] A. Bemporad, M. Morari, V. Dua and E.N. Pistikopoulou, "The Explicit Linear Quadratic Regulator for Constrained Systems", In Proceedings on Control Conference, pp. 872-876, 2000
- [25] P. Dorato, C. Abdallah and V. Cerone, "Linear-Quadratic Control, An Introduction," Prentice Hall, Englewood Cliffs, New Jersey, 1995.
- [26] K. Ogata, "Modern Control Engineering," Prentice Hall, Upper Saddle River, New Jersey, 2010.
- [27] T. Kailath, "Linear Systems," Prentice-Hall, Englewood Cliffs, New Jersey, 1980.
- [28] C.E. Baumgartner, H.P. Geering, C.H. Onder and E. Shafal, "Robust Multivariable Idle Speed Control," American Control Conference (ACC), pp. 258-265, June 1986.

Biographies



Simon O. Omekanda was born in Rabat, Morocco, on September 15, 1984. He received his Bachelor in Science (B.S.) degree in electrical engineering from Pennsylvania State University, State College, in 2009 and the Masters in Science (M.S.) degree in electrical and computer engineering from Oakland University, Rochester, MI, in 2011. He is currently getting his Ph.D in controls engineering from Oakland University. He has been working as a teaching and research assistant for the electrical and computer engineering department at Oakland University during this time. He has taught and helped students in the areas of controls and estimation theory as well as using matlab/simulink for simulation. His research areas primarily include design, implementation and simulation of nonlinear observers for state estimation and control, multivariable control, subsystem interconnection with application to internal combustion engines and electric machine in automotive applications. He has published one paper titled "Application of Extended Observer with Gain Scheduling Control to Internal Combustion Engine Model" in a similar area as the one presented here.



Mohamed A. Zohdy was born in Caro, Egypt. He received B.Sc. degree in Electrical Engineering, Cairo University, Egypt in 1968. He received his M.S. and Ph.D. degrees in Electrical Engineering, University of Waterloo, Canada in 1974, and 1977 respectively. He is currently a Professor at Oakland University, Rochester Hills, Michigan, U.S.A. His research interests are in areas of Controls and Soft Computing.







A Two Branch SIMULINK Model of the Lithium Polymer Battery

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Abstract

The paper discusses the Simulink model of lithium polymer battery. Model parameters are temperature and state of charge dependent. This model is two branch (parallel R-C) model. 2-D Look-up table method is used to represent the relation between model branch parameters and state of charge. Matlab user defined function is used to calculate the open circuit voltage and internal dc resistance. One whole block is dedicated to state of charge calculation. This block is able to autonomously detect the transition from charge to discharge and vice versa. Also if charging or discharging current stops flowing, it resets the current integrator initial condition to the last state of charge calculated. Comparison between experimental and model results showed about 3% maximum error in dynamic (interval test) response and 5% error in static response of battery discharging (until 10% state of charge is left in the battery).

Keywords: Battery, look-up table, state of charge.

Nomenclature

PSCAD	Power System Computer Aided Design
R-C	Resistor- Capacitor
SOC	State Of Charge
DAQ	Data Acquisition
A/D	Analog to digital
D/A	Digital to analog
AH	Amp-Hour
V_t, v	Terminal voltage
V_{oc}, E	Open circuit voltage
R_{dc}, R_0	Internal DC resistance
R_1, R_s	Resistance of short time constant
R_2, R_l	Resistance of long time constant
C_1, C_s	Capacitance of short time constant
C_2, C_l	Capacitance of long time constant
I	Current
t	Time
T	Constant time interval
k	Constant of temperature directly proportional to temperature (Table-5)
a_m	Coefficients of quadratic equation (Table-6)
Q	Battery capacity in amp-sec (AS)

1 Introduction

Increasing the throughput of electric vehicles (EV) is very important. To do that, main component of the EV, the battery, must be tested, analyzed and modeled before it can be installed in EV. Dynamic models of batteries are important for analysis, design and performance of EV [1-3]. Most models developed until now are for small batteries (low capacity), used in cell phones, computers, and portable electrical appliances which show less nonlinearity. But electric vehicle grade batteries (high capacity) are highly nonlinear in nature. The surface kinetics, thermal behavior, reaction rate of big batteries differs from small batteries.

Proposed model is adopted from [4] (which is developed based on the experiments on batteries used for laptop computers, PDAs, mobiles etc.) and it is based on equations, so it can be implemented in different simulation platforms as it has been implemented in PSCAD [5-6]. From all the models [1-23], model from [4] showed the best accuracy in dynamic as well as static characteristics. For the development of model presented here, each branch voltage (voltage across dc resistance, voltage across each pair of R-C, open circuit voltage) calculation is divided into blocks. Developing a model in different blocks is to identify different function of each block that develops the model. Each block represents contribution to terminal voltage calculations. These block parameters depends on state of charge (SOC) and temperature. The parameters are polynomial equations depending on SOC and temperature [7].

Wide variety of models has been developed by researchers with varying degree of complexity. Electro-chemical models developed in [8-10] mainly designed for optimization of physical design of the battery. But these models involve complex time-variant partial differential equations which require days to simulate and requires complex algorithms. Mathematical models [11-13] lacks in practical meaning but useful to system designers, uses mathematical methods like stochastic approaches [12] to predict battery runtime, efficiency or capacity. However, mathematical models do not give out the SOC-V information that is important in circuit simulation and optimization. Moreover, mathematical models provide inaccurate results in the order of 5%-20% error. Electrical models [1-7, 14-23] provides the accuracy



somewhere in between electro-chemical models and mathematical models (around 1%-5% error). Following few paragraphs describes few of the developed electrical models.

In figure (1), a Thevenin's equivalent circuit model is presented, which has been extensively used to model the discharging dynamics of the batteries such as Lead acid, Lithium polymer, Lithium-Ion (Li-Ion) etc. [1-3,5-7]. The easiest model is the first order Thevenin's model with one parallel R-C branch and its parameters can be easily estimated experimentally. The R-C combinations predict the response to a transient load at specific SOC's by assuming that the open circuit voltage is constant. Therefore, this model is unable to reflect the influence of the SOC to the battery behavior properly. Taking this into consideration the battery voltage response might be different from first order model response [14]. So, higher order model is required to capture the behavior of the battery. For some batteries second order model is sufficient e.g lithium polymer batteries.

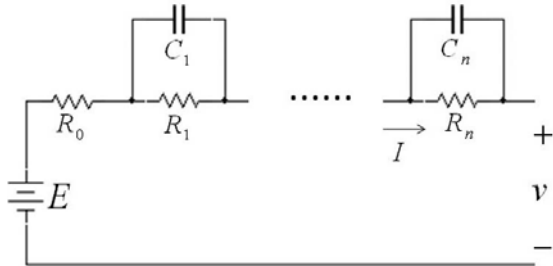


Figure 1: Thevenin's equivalent model for discharging battery [14]

An impedance based circuit acquires an AC-equivalent impedance model in the frequency domain. It then uses a complex equivalent system (Z_{ac}) to fit the impedance ranges. This fitting process is very difficult and complex. Additionally, since these models only work for fixed SOC's and temperatures, they cannot predict battery runtime or DC response [15].

Runtime based models use complex circuits to simulate the DC voltage response and runtime of the battery. This model used two different circuits to depict the battery characteristics. One circuit depicts a capacitor, which contains the value of the battery capacitance, and a dependent current source which represents the batteries state of charge. And the second circuit consists of the R-C networks which represent the relationship between the battery current and terminal voltage [4]. This circuit is similar to the Thevenin's equivalent circuit. But these models do not take into consideration the changing behavior of passive elements with varying SOC.

Developed model incorporates the effect of SOC and temperature on different parameters of the battery model. Also, the model is valid for different chemistries provided the equations for different parameters of the battery depending on SOC and temperature are available, but at the time model is tested for Li-Ion chemistry.

The remainder of the paper is organized as follows: section (2) describes the battery evaluation lab, section (3) focuses on test system and test method to extract the parameters of the battery model, section (4) emphasizes on the development of the model in MATLAB/SIMULINK, section (5) emphasizes on SOC calculation block, section (6) analyses and compares the

results obtained from the simulation to the experimental results and section (7) concludes the paper with emphasizing the contribution of the paper in developing the battery model with parameters depending on SOC and temperature.

2 Battery Evaluation Lab

The tests on the batteries are carried out at the battery evaluation lab at University of Massachusetts Lowell. The lab is equipped with current regulators controlled by computer. There are two types of regulators.

1. 8 current regulators with capability of upto 10V and 320 amps source or sink.
2. 4 current regulators with capability of upto 20V and 160 amps source or sink.

All the current regulators are capable of going from charging to discharging instantaneously [16] and are controlled via A/D and D/A controllers, which gets the commands from data acquisition (DAQ) system controlled by computer. All the programs are written in BASIC. Different current profiles can be easily written with BASIC programming language. You can load external current command file, so that realistic load tests can be done.

Lab is also equipped with two environmental chambers to test the batteries at different temperatures. Range of temperature control is from -20°C to +50°C. Before every test the battery is soaked to ambient temperature. If the temperature is 0°C or below battery is soaked overnight to make sure that center of the battery is at desired temperature.

3 Test system and test method

Test system is shown in figure (2). As it is seen, the battery which resides in the environmental chamber is connected to the current regulators and MUX. Current regulators and environmental chamber are controlled through computer via DAQ.

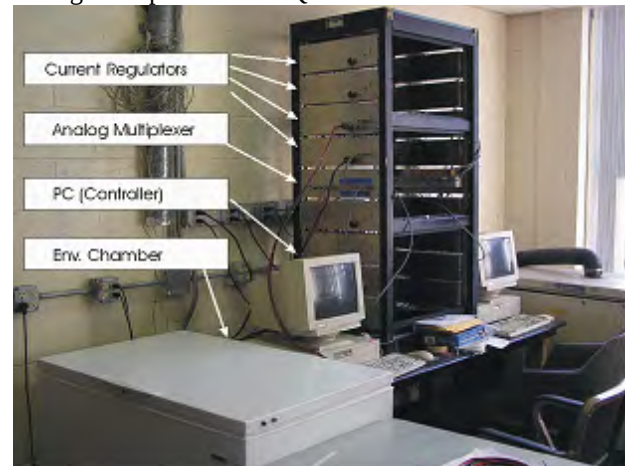


Figure 2: Test setup

The battery model must resemble the dynamic and static characteristic of battery behavior in real world. So, two types of tests are done to extract the parameters of battery model. Interval test is done to monitor the battery dynamics and provides information about the short time and long time constant parameters for the battery model.



Constant discharge test is done to obtain the DC resistance.

Interval test is constant discharge followed by rest. Constant discharge in interval test is pre specified and set to 10Ah (0.1 SOC). The rest period is set to 10 minutes so that by that period the dynamic effect of short time and long time constants is stabilized and battery reaches to open circuit voltage. Figure (3) shows the interval discharge at 0°C and 100 amps. For constant discharge test, battery is subjected to constant current of 100amps at different temperature. Figure (4) shows the constant discharge at 20°C and 100 amps.

Presented model is developed for discharging battery, but the same tests (interval test and constant discharge test, explained in detail later in the paper) can be done to get the parameters of the battery during charging and can be incorporated in the same model. Same parameters are used for charge and discharge for this model.

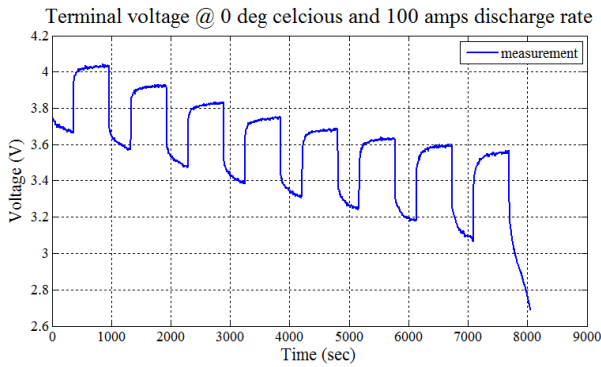


Figure 3: interval discharge test done at 0°C and 100 amps

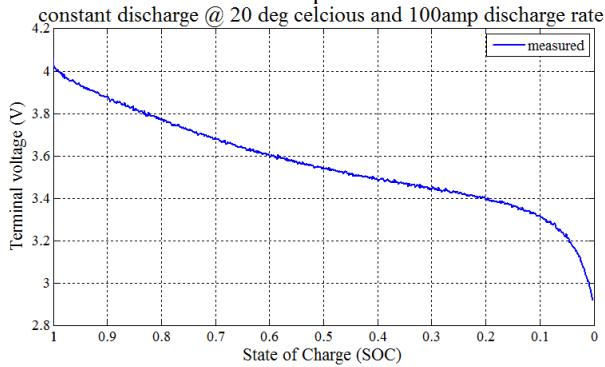


Figure 4: constant discharge test done at 20°C and 100 amps

To be able to predict the behavior in different weather condition tests are done at different temperature settings. The temperature range is from 0° to 40° C.

Parameter extraction: Thevenin second order circuit model solution is given by:

$$V_t = V_{oc} - IR_{dc} - IR_1 \left(1 - e^{-\frac{t}{R_1 C_1}}\right) - IR_2 \left(1 - e^{-\frac{t}{R_2 C_2}}\right) \quad (1)$$

The method used to extract the parameters for model is adopted from [14]. It's a simple analytical method. In this paper explicit formulas are used to compute the n th-order model. But we used its 2nd order model derivation. We tweaked the results of [14] to work for interval test. Instead of being able to work at particular SOC beginning, we added a loop that at any instance of time if

current goes to zero and after some time discharging current starts to flow again program recalculates the parameters with new values of (v, t) specified at constant time intervals T. Tables 1-4 show the parameters extracted at different temperature.

Figure (5) and figure (6) shows the graphs of open circuit voltage and DC resistance at different temperature respectively with both the measurement and simulation results.

Table 1 R_s -SOC values at different temperatures

SOC	Temperature (°C)		
	0	20	40
0	0.00086	0.0006	0.00016
0.1	0.000722	0.00025	0.000122
0.2	0.000621	0.00022	0.000121
0.3	0.00062	0.00022	0.00012
0.4	0.00050061	0.00020061	0.00010061
0.5	0.000222291	0.000162291	0.000102291
0.6	0.00020408	0.00013408	0.00010408
0.7	0.00018078	0.00010078	0.00010078
0.8	0.00017836	0.00025836	0.00011836
0.9	0.000203084	0.000063084	0.000103084
1	0.00014976	0.00012976	0.00008976

Table 2 C_s -SOC values at different temperatures

SOC	Temperature (°C)		
	0	20	40
0	20000	56000	60000
0.1	22011	40011	60011
0.2	26379	40379	60379
0.3	31000	71000	61000
0.4	51696	81696	61696
0.5	57680	107680	67680
0.6	54089	44089	64089
0.7	55760	225760	65760
0.8	57760	187760	67760
0.9	56265	66265	66265
1	52847	202847	62847

Table 3 R_i -SOC values at different temperatures

SOC	Temperature (°C)		
	0	20	40
0	0.00061	0.02	0.00026
0.1	0.00057	0.02	0.00025
0.2	0.00053	0.00044	0.00023
0.3	0.00049	0.00047	0.00022
0.4	0.00047	0.00047	0.00027
0.5	0.00042	0.00045	0.00025
0.6	0.0003	0.00039	0.0001
0.7	0.00016	0.00023	0.0001
0.8	0.00017	0.00007	0.00017
0.9	0.00015	0.000155	0.00017
1	0.00016	0.00017	0.0001



Table 4 C_1 -SOC values at different temperatures

SOC	Temperature ($^{\circ}\text{C}$)		
	0	20	40
0	249350	249350	249350
0.1	314350	164350	314350
0.2	329880	419880	419880
0.3	352000	402000	402000
0.4	401950	401950	401950

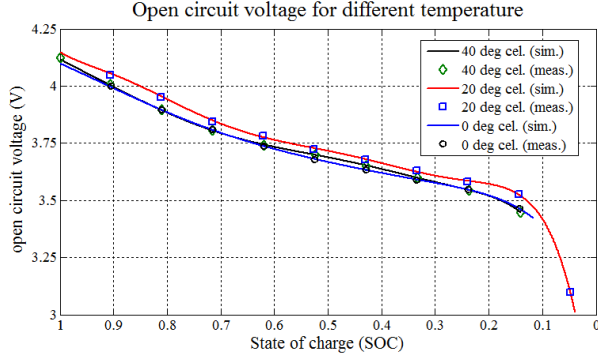


Figure 5: Open circuit voltage of the battery at different temperature with measurement and simulation

0.5	401240	401240	401240
0.6	401660	401660	401660
0.7	401870	401870	401870
0.8	408170	408170	408170
0.9	409930	409930	409930
1	408920	508920	408920

In the interval discharge test open circuit voltage (V_{oc}) and internal dc resistance (R_{dc}) is recorded at each end of rest period:

$$V_{oc} = v(0^-) \quad (2) \quad R_{dc} = v(0^-) - v(0^+) / I \quad (3)$$

After finding V_{oc} and R_{dc} at different SOC points, polynomial equation is derived using curve fitting toolbox to relate V_{oc} and R_{dc} with SOC. With three different temperatures we have three different equations for V_{oc} and three different equations for R_{dc} . Combining these equations produced following equations:

$$V_{oc} = \sum_{n=0}^7 b_n soc^n \quad (4) \quad R_{dc} = \sum_{n=0}^8 b_n soc^n \quad (5)$$

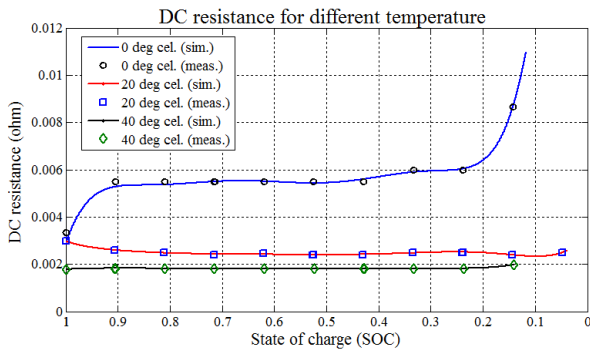


Figure 6: DC resistance of the battery at different temperature with measurement and simulation

Table 5 Relation between temperature and constant k

Temperature ($^{\circ}\text{C}$)	k
0	0
20	2
40	4

Where,

$$b_n = \left\{ \sum_{m=0}^2 (a_m k^m) \right\}_n$$

k and a_m values are presented in table 5 and table 6 respectively

At 0°C the dc resistance rises very quickly toward the end of discharge. Also values of dc resistances at different SOC are relatively higher than 20°C and 40°C , which happens to be due to slow kinetics at low temperatures. As temperature increases the value of dc resistance decreases.

4 Simulation model

The model is implemented in MATLAB/SIMULINK simulation environment. Visualization of equations becomes so easy with Simulink's graphical presentation. User defined function written in Matlab script editor can be used in SIMULINK.

This model is two circuit model connected through nonlinear voltage controlled voltage source and a current controlled current source. The first circuit consists of big capacitor representing SOC of the battery and the other circuit is two pairs of parallel R-C branch connected in series with internal dc resistance [17] as shown in figure (7).

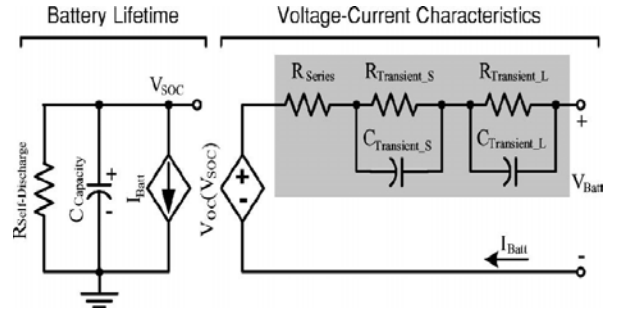


Figure 7: Proposed battery model adopted from [4]

Self-discharge resistance is not considered in the circuit as the battery is subjected to frequent use. R_{dc} and V_{oc} can be found from constant discharge test, but it is only possible for initial condition e.g if the constant discharge test starts at 90% SOC than R_{dc} and V_{oc} at 90% SOC is available. But to establish the relationship of R_{dc} and V_{oc} with SOC, interval test is done. From this test other component values are obtained such as R_s , C_s and R_l , C_l components of short time constant and long time constant respectively. Battery behaves differently under different climates, so relationship of R_{dc} , V_{oc} , R_s , C_s , R_l , C_l with SOC changes at different temperatures. All the model components and SOC are related by polynomial equation. So while the model is running, model calculates the values of all the components dynamically and uses it to find the terminal voltage at that particular SOC. So overall all the calculations depend on SOC of the battery. One block is allocated for the calculation of SOC, which takes care of following actions:

1. Sudden start of charge from discharge and vice-versa
2. Rest after charge or discharge.

During these conditions the integrator resets itself to new initial condition.



Table 6 Values of coefficients a_2 , a_1 , a_0 of different b , which are coefficients of V_{oc} and R_{dc}

	V_{oc}			R_{dc}		
	a_2	a_1	a_0	a_2	a_1	a_0
b_8	-	-	-	0.01194875	-0.1033125	0
b_7	7.3705	-14.741	0	-0.05989975	0.4332045	-0.075432
b_6	-29.075	77.1571	-42.848	0.10599375	-0.6961025	0.22984
b_5	48.6737	-167.355	153.79	-0.07906375	0.5003425	-0.20603
b_4	-45.1448	192.389	-219.71	0.017924363	-0.1190662	0.0082249
b_3	25.0809	-125.036	159.4	0.00658875	-0.035942	0.075254
b_2	-8.2449	45.3	-61.088	-0.00388465	0.02335445	-0.035913
b_1	1.4449	-8.3407	11.983	0.00077195	-0.0046413	0.0083023

All the equations for the model of discharging batteries are as follows:

$$SOC(t) = SOC(0) - \int_0^t \frac{1}{Q} x(t) dt \quad (6)$$

Where,

$SOC(0)$ =initial SOC

$$V_t = V_{oc} - IR_{dc} - IR_s \left(1 - e^{-\frac{t}{R_s C_s}}\right) - IR_l \left(1 - e^{-\frac{t}{R_l C_l}}\right) \quad (7)$$

In the figure (8), source (supply) named “current” can be set to run in constant discharge mode or interval discharge mode. Charging can also be incorporated in the same source. This current is fed to four subsystems namely SOC (scion colored block), Internal DC resistance (grey colored block), short time constant (green block), and long time constant (orange block). First the SOC block is executed. Value of temperature is fed to 1-D look up table called k and also other 2-D look up tables R_s , C_s , R_l and C_l . Using SOC and temperature

values of R_s , C_s , R_l and C_l are determined, and fed to short and long time constant blocks. V_{oc} and R_{dc} are written in user defined matlab function. Values of k and SOC are supplied to this function to evaluate the equations. V_t is determined in four parts. The four parts are as follows:

1. Open circuit voltage V_{oc}
2. Internal DC resistance block is the mathematical representation of IR_{dc}
3. Short time constant block is the mathematical representation of $IR_s \left(1 - e^{-\frac{t}{R_s C_s}}\right)$
4. long time constant block is the mathematical representation of $IR_l \left(1 - e^{-\frac{t}{R_l C_l}}\right)$

To get the value of v_t parts 2, 3 and 4 are subtracted from 1. Two conditions are applied if the voltage goes above 4.2V or voltage goes below 2.7V the model stops execution.

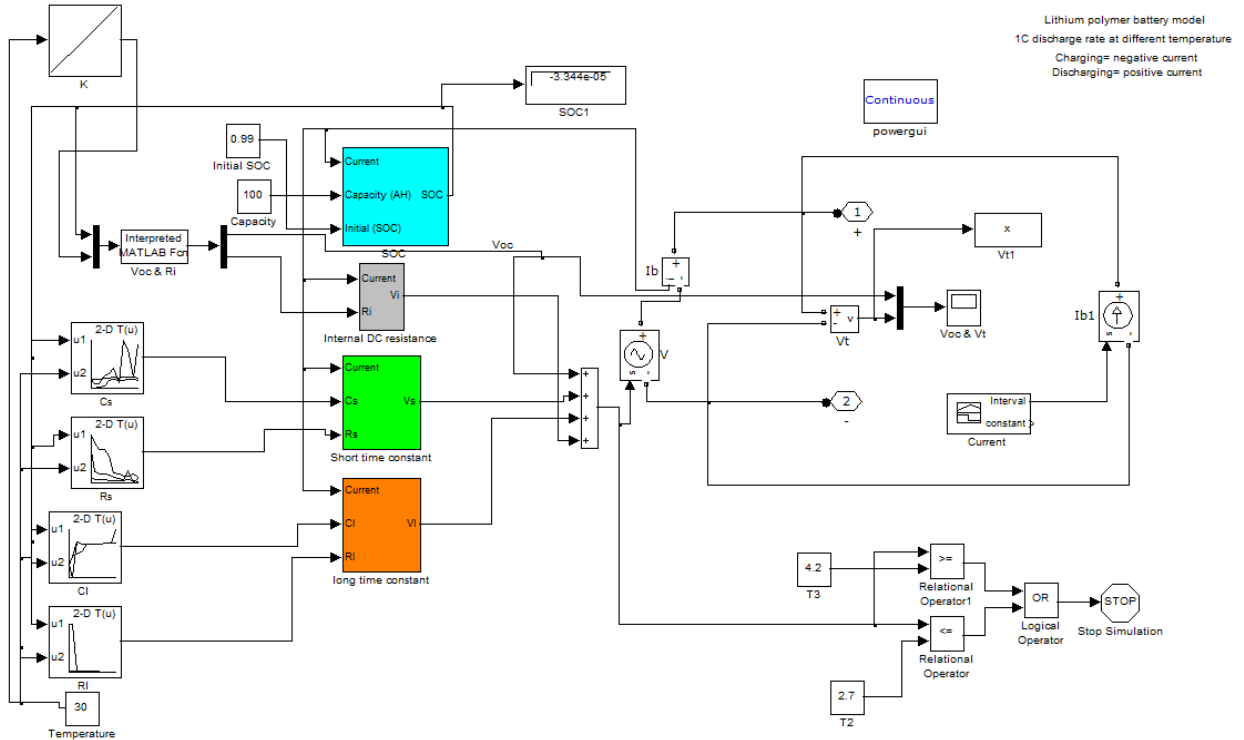


Figure 8: Simulation model of lithium battery



5 SOC calculation

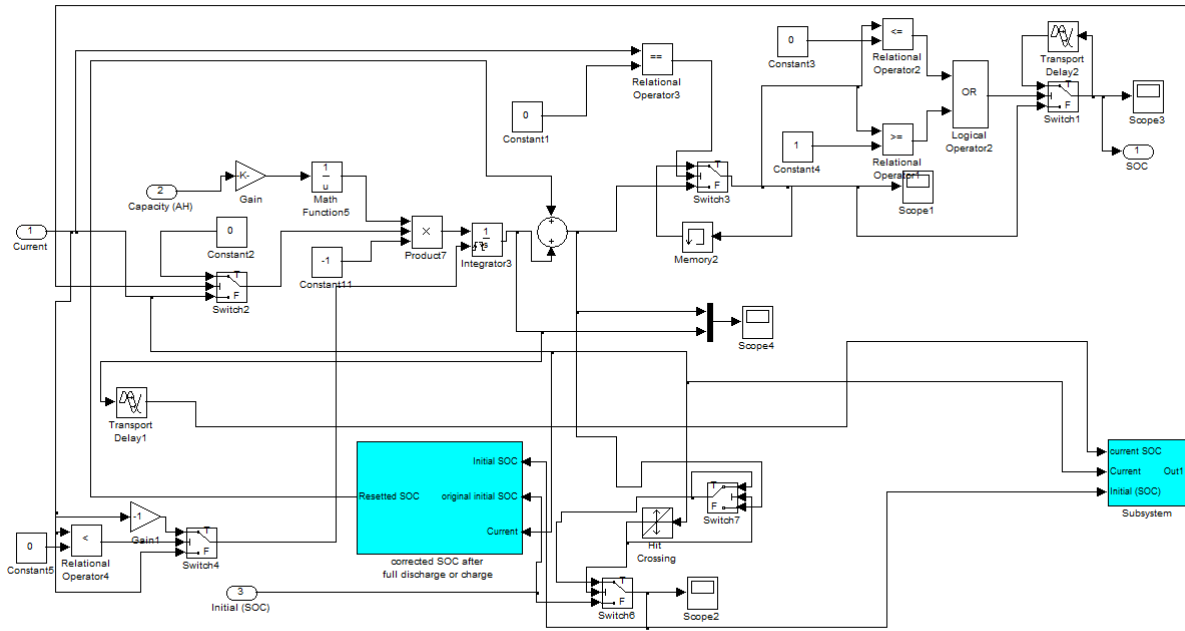


Figure 9: Subsystem block SOC

There are three modes of operation during which SOC varies accordingly,

1. charge mode, SOC increases during this mode
2. discharge mode, SOC decreases during this mode
3. rest mode, SOC remains unchanged during this mode

SOC calculation is a big part of this model and it is done in SOC block (figure (9)) of the model. Battery capacity, initial SOC and current are input to this model and SOC is the output. First the capacity is converted into amp-sec (AS) format from amp-hour (AH) and inverted to be multiplied by current. Reset value of SOC is added to compensate for full discharge or charge condition as well as transition from one mode to another. So if battery is discharging and SOC reaches to 0 the output of the adder is set to 0. And if the battery is charging and SOC reaches to 1 the output of the adder is set to 1. Reset value of SOC comes from block named “corrected SOC after full discharge or charge”. If the battery is in discharge mode and it reaches to 0 SOC but the battery does not go in charge or rest mode than this block holds the 0 SOC, same goes for charge process. But this block does not have to hold these conditions for long because at 0 SOC output voltage is 2.7V and at 1 SOC output voltage is 4.2V so, simulation stops. Also this block works as memory i.e, if battery goes into rest mode or charging mode from discharge mode, this block memorizes the value of SOC at the time of transition and uses it as initial SOC for next mode. After the addition block one more condition is checked if the current is zero than the same value of SOC is maintained.

6 Results

For the interval test current value has to be equal for all temperatures so that we don't have to change the time value for discharge during each interval. Figure (10) shows the current profile for interval discharge test. Positive current is for discharge and negative current represents charging process. Interval test was done at different temperatures.

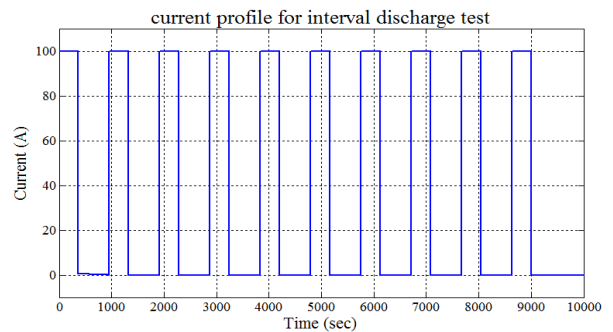


Figure 10: current regime for interval discharge test

The current profile is chosen such that battery discharges for 10% SOC and then enters into rest mode. For tested battery of 100Ah 10% SOC corresponds to 10Ah. So if the battery is discharged at 100 amps current then the interval should be of 360secs to discharge the battery 10Ah. Figure (11 (a)) and figure (11(b)) shows the terminal voltage comparison between measurement and simulation of interval tests at 0°C and 40°C, 100amps respectively. Figure (12) shows the terminal voltage results of measurement and simulation of constant current discharge tests at 0°C and 40°C, 100amps respectively.



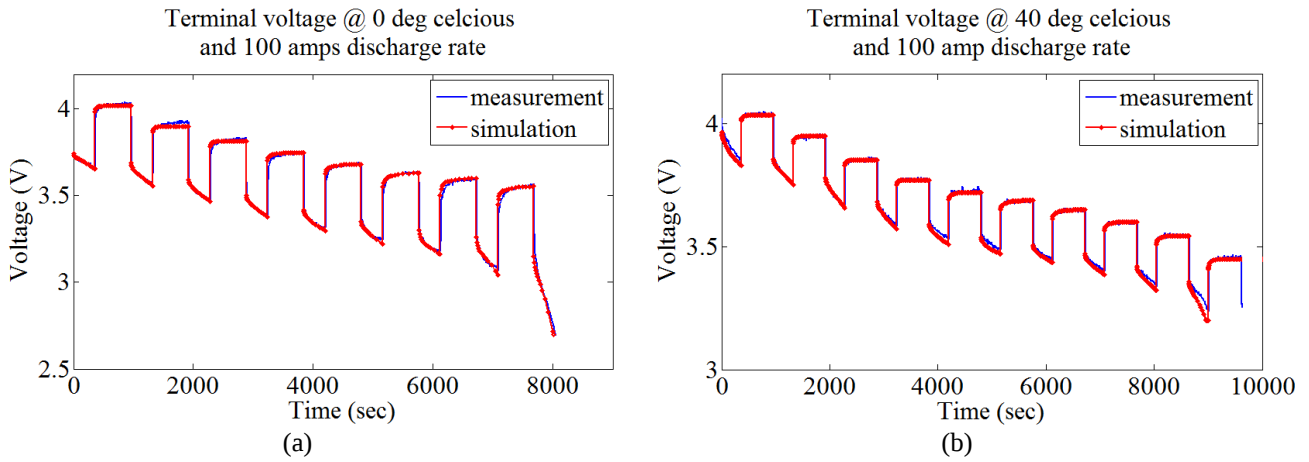


Figure 11: Terminal voltage of the battery subjected to transients of the interval of 10% SOC followed by 10 min rest (a) at 0°C and 100 amps discharge rate (b) at 40°C and 100 amps discharge rate.

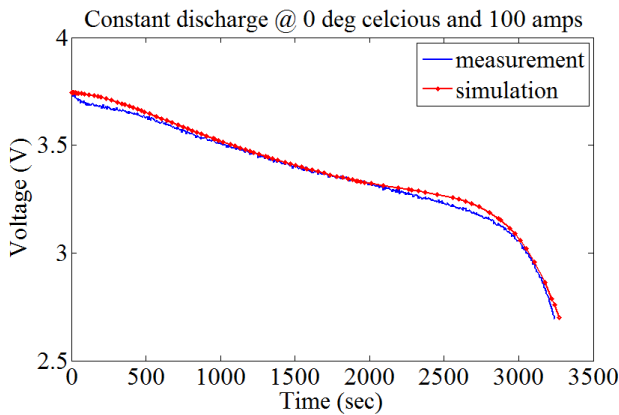


Figure 12: Terminal voltage of the battery subjected to constant discharge at 0°C and 100 amps discharge rate

The close agreement between experimental and simulation result proves the accuracy of extracted circuit parameters and indirectly implies that the parameters depend the SOC and temperature. The runtime error between measurement and simulation are merely 1% and 1.2% for interval test and constant discharge test respectively at 0°C and 100amps. The maximum error occurred was when battery was subjected to 40°C and 100 amps discharge in both discharge tests (interval test and constant discharge test). At the end of the discharge when 20% SOC is left in the battery, it started behaving abnormally. The error at the end of discharge is 62mV (below 20% SOC). Above 20% SOC the maximum error in interval test is 40mV. The reason behind that might be that after the initial temperature of 40°C while the battery was discharging, the temperature increased to a level that electrolyte could not handle. Below 0°C the battery does not behave normally, so tests below 0°C are avoided. At lower temperature terminal voltage of the battery dips at the start of the discharge but after some time it jumps back to its normal state, as shown in figure (12).

7 Conclusion

The detailed model of Lithium batteries can play a vital role in battery management system developed for use in Electric Vehicles (EVs). This model is two R-C branch model instead of three and still has good accuracy. Precise SOC calculation is done to provide accurate values of open circuit voltage, internal DC resistance,

components of short term time constant and long term time constant. The polynomial equations of open circuit voltage and DC resistance are of the 7th and 8th order respectively to capture the precise value. This model is designed for both charge and discharge. This model makes use of tabular method to find values of components of short term and long term time constants depending on SOC. Interpolation and extrapolation capabilities of the table used in Simulink help in finding values other than test values.

Results show that the maximum error between model and experimental data between 100%- 20% SOC is about <1% for interval test and 1.2% for constant discharge test. But at the end of simulation (20%-0% SOC) this error increases. Because the interval test and constant discharge tests are done at different open circuit voltage, error in the terminal voltage is high for constant discharge test. At lower temperatures dc resistance increases unexpectedly at low SOC.

References

- [1] S. X. Chen, K. J. Tseng, and S. S. Choi, "Modeling of lithium-ion battery for energy storage system simulation," in Proc. Power Energy Eng. Conf., 2009, pp. 1–4.
- [2] C. Speltino, D. Di Domenico, G. Fiengo, and A. Stefanopoulou, "Comparison of reduced order lithium-ion battery models for control applications," in Proc. Joint IEEE 48th Conf. on Decision Control and Chinese 28th Control Conf., 2009, pp. 3276–3281.
- [3] S. Tian, M. Hong, and M. Ouyang, "An experimental study and nonlinear modeling of discharge IV behavior of valve-regulated lead-acid batteries," *IEEE Trans. Energy Convers.*, vol. 24, no. 2, pp. 452–458, Jun. 2009.
- [4] Min Chen; Rincon-Mora, G.A.; , "Accurate electrical battery model capable of predicting runtime and I-V performance," *Energy Conversion, IEEE Transactions on*, vol.21, no.2, pp. 504- 511, June 2006.



- [5] Yessayan G.; "A modeling of a Lithium Iron Phosphate battery using PSCAD", Master thesis, UMass Lowell, Apr. 2012.
- [6] Sharma S., "modeling of a Lithium polymer battery using PSCAD", Master thesis, UMass Lowell, Apr. 2012.
- [7] Prieto, R.; Oliver, J.A.; Reglero, I.; Cobos, J.A.; , "Generic Battery Model based on a Parametric Implementation," *Applied Power Electronics Conference and Exposition, 2009. Twenty-Fourth Annual IEEE*, vol., no., pp.603-607, 15-19 Feb. 2009.
- [8] L. Song and J. W. Evans, "Electrochemical-thermal model of lithium polymer batteries," *J. Electrochem. Soc.*, vol. 147, pp. 2086–2095, 2000
- [9] D. W. Dennis, V. S. Battaglia, and A. Belanger, "Electrochemical modelling of lithium polymer batteries," *J. Power Source*, vol. 110, no. 2, pp. 310–320, Aug. 2002.
- [10] J. Newman, K. E. Thomas, H. Hafezi, and D. R. Wheeler, "Modeling of lithium-ion batteries," *J. Power Sources*, vol. 119–121, pp. 838–843, Jun. 2003.
- [11] R. Rynkiewicz, "Discharge and charge modeling of lead acid batteries," in *Proc. Appl. Power Electron. Conf. Expo.*, vol. 2, 1999, pp. 707–710.
- [12] D. Rakhmatov, S. Vruthula, and D. A. Wallach, "A model for battery lifetime analysis for organizing applications on a pocket computer," *IEEE Trans. VLSI Syst.*, vol. 11, no. 6, pp. 1019–1030, Dec. 2003.
- [13] P. Rong and M. Pedram, "An analytical model for predicting the remaining battery capacity of lithium-ion batteries," in *Proc. Design, Automation, and Test in Europe Conf. and Exhibition*, 2003, pp. 1148–1149.
- [14] Tingshu Hu; Zanchi, B.; Jianping Zhao; "Simple Analytical Method for Determining Parameters of Discharging Batteries," *Energy Conversion, IEEE Transactions on*, vol.26, no.3, pp.787-798, Sept. 2011
- [15] "Study of Battery Modeling Using Mathematical and Circuit Oriented Approaches." *Study of Battery Modeling Using Mathematical and Circuit Oriented Approaches*. IEEE. Web. <<http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=06039230>>.
- [16] Lynch, W.A.; Salameh, Z. M., "Electrical component model for a nickel-cadmium electric vehicle traction battery," *Power Engineering Society General Meeting, 2006. IEEE*.
- [17] Knauff, M.C.; Dafis, C.J.; Niebur, D.; Kwatny, H.G.; Nwankpa, C.O.; , "Simulink Model for Hybrid Power System Test-bed," *Electric Ship Technologies Symposium, 2007. ESTS '07. IEEE*, vol., no., pp.421-427, 21-23 May 2007.
- [18] Lijun Gao; Shengyi Liu; Dougal, R.A.; , "Dynamic lithium-ion battery model for system simulation," *Components and Packaging Technologies, IEEE Transactions on*, vol.25, no.3, pp. 495- 505, Sep 2002.
- [19] Z. M. Salameh, M. A. Casacca, and W. A. Lynch, "A mathematical model for lead-acid batteries," *IEEE Trans. Energy Conversion*, vol. 7, no. 1, pp. 93–98, Mar. 1992.
- [20] Singh, P.; Nallanchakravarthula, A.; , "Fuzzy logic modeling of unmanned surface vehicle (USV) hybrid power system," *Proceedings of the 13th International Conference on Intelligent Systems Application to Power Systems*, vol., no., pp.7 pp., 6-10 Nov. 2005.
- [21] Saiju, R.; Heier, S.; , "Performance analysis of lead acid battery model for hybrid power system," *Transmission and Distribution Conference and Exposition, 2008. T&D. IEEE/PES*, vol., no., pp.1-6, 21-24 April 2008.
- [22] Knauff, M.; Niebur, D.; Nwankpa, C.; , "A platform for the testing and validation of dynamic battery models," *Electric Ship Technologies Symposium, 2009. ESTS 2009. IEEE*, vol., no., pp.554-559, 20-22 April 2009.
- [23] Kroeze, R.C.; Krein, P.T.; , "Electrical battery model for use in dynamic electric vehicle simulations," *Power Electronics Specialists Conference, 2008. PESC 2008. IEEE*, vol., no., pp.1336-1342, 15-19 June 2008.

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Hybrid Dynamic System Control of Sequential Motion in Biped Robots

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Abstract

This paper presents a new hybrid dynamic approach to describe and control sequential biped robot motions including sitting and standing. Transitions between these motions are expressed as a hybrid dynamic system that express each pose of the biped. Control of the biped in each state is performed using nonlinear backstepping with integral action. Stability and robustness of the ensuing biped robot control are addressed. Simulations are used to further demonstrate the effectiveness of control.

Keywords: nonlinear control, integrator backstepping, hybrid dynamic systems, biped robot.

Nomenclature

q_i	Discrete states of a hybrid system
x	Continuous-time state space of a hybrid system
D	Domain of continuous states in a discrete state q
f	Function describing the continuous dynamics in a state q .
E	Edges, possible transitions from states
G_{ij}	Guards, conditions that must be met to transition to another state
R	Resets, reset maps between states
$D(q)$	Inertia matrix of a robot, size of $n \times n$.
$H(q, \dot{q})$	Friction, coriolis and centripetal forces, size of $n \times 1$.
$G(q)$	Gravitational forces acting on each link's center of gravity, size of $n \times 1$.
u	Actuator torques acting on each link, size of $n \times 1$.
S_1	HDS describing a sitting biped
S_2	HDS describing a standing biped
$c_1, \dots, c_{2n}$	Backstepping controller parameters for an $n \times n$ robot.
$k_1, \dots, k_n$	Backstepping controller integral gains for an $n \times n$ robot.

1. Introduction

There are several aspects to walking bipedal motion including the multi degree of freedom dynamic model of the biped [1], control and coordination of the walking gait [2]-[4], actuator control, and disturbance rejection because of terrain and other external forces. Each of these topics is very important by itself and combining them together to form a working system is a continuing challenge.

In this paper, a framework for hybrid dynamic systems (HDS) is presented to show that it is desirable to organize biped robotic motions into multiple levels of subsystems for control. Organizing biped motions into hybrid dynamic systems, sometimes with a reduced model order, allows for greater understanding when controlling transitions such as sitting and standing or between standing and sitting. Examples are shown by using 1-link and 2-link representations of a sitting biped robot model derived using Lagrangian equations of motion [5].

Around the hybrid states the biped is controlled using nonlinear backstepping control. This method of control has several benefits including robustness against unmodeled dynamics and meeting stability and performance requirements [6]-[12]. A scheme for 2-link robotic actuators will be applied to the biped system. Each HDS has a unique controller and the transitions between HDS systems require the controller's smooth transfer under predetermined conditions. Sequencing of HDS states and control is shown through examples of motions from a sitting biped to a standing biped. Implementations of the simulations of models are performed in Matlab/Simulink. Simmechanics is used to model the biped robot. Zhao and Liu proposed a 3D Simmechanics model of a biped for simulation which was capable of using joint angle actuator drivers to simulate the kinematics of the biped [13]. However, their model is not able to simulate dynamic feedback control. In this paper a simplified 2D Simmechanics model is presented that is capable of simulation with actuator torque for dynamic feedback control.

While hybrid dynamic systems and backstepping use is well documented in many applications, both have not been combined and used extensively in biped robotics. In this paper a HDS system and transitions are designed to describe biped robot motions. In section 2, an example of HDS definitions for sitting and standing motions is



given. Section 3 provides the biped model derivation using Lagrange Equations. In section 4, backstepping controllers with integral action are designed for sitting and standing states. Section 5 shows simulations of the sitting to standing events, compares the backstepping control to a classical PD control [14] and explores stability and robustness. Section 6 is a conclusion of the work of this paper.

2. HDS Model Of Action Motions

Hybrid Dynamic Systems contain both continuous dynamics and overlaid discrete events. Within a hybrid state, the continuous dynamics are described by a system of differential or difference equations. While in a hybrid state the dynamics flow in time but can transition to the same state or to another state when certain guard conditions or constraints are met. Jumps to other hybrid states may cause state variables to reset based on defined jump event maps. The order of continuous dynamics in each hybrid state may also vary and the maps will define how this change and its effects are handled. This notation of an HDS system is known as the hybrid automaton of the form $S = (Q, X, D, f, E, G, R)$ as shown in Figure 1 [15]- [17].

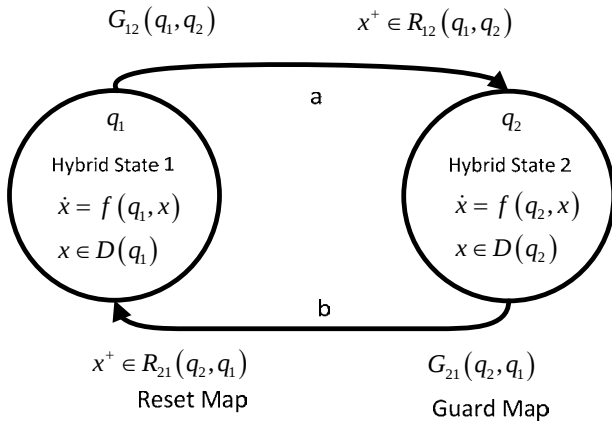


Figure 1. Hybrid Dynamic System model with two hybrid states.

The variables in Figure 1 are defined as follows.

q_i – Discrete states of the hybrid system (mode of the system); e.g. falling, rising, accelerating, decelerating.
 x – Continuous-time state space of the hybrid system.
 D – Allowable domain of continuous states in a discrete state q .

f – Vector field function describing the continuous dynamics, often referred to as the flow, of x in a state q .

E – Edges, possible transitions from states such as from q_1 to q_2 , edges in Figure 1 are labeled “a” and “b”.

G_{ij} – Guards, conditions that must be met at an edge to transition to another state.

R_{ij} – Resets, reset maps to define how state variables change when transition events from one state to another occur, such as from q_1 to q_2 .

Reduced ordered models are used here to simplify the modeling and control of the biped robot. The sequence of motions are divided into action motions, each described by a hybrid dynamic system. For example, while in a sitting position the biped is modeled as a single link, an inverted pendulum. The torso angle is regulated to provide correct sitting postures while other actuators are left unactuated. When a transition to a standing position is desired the controller then attempts to reach that desired position needed to propel the biped into a desired standing position. The HDS for action motions in this paper are described below in Figure 2.

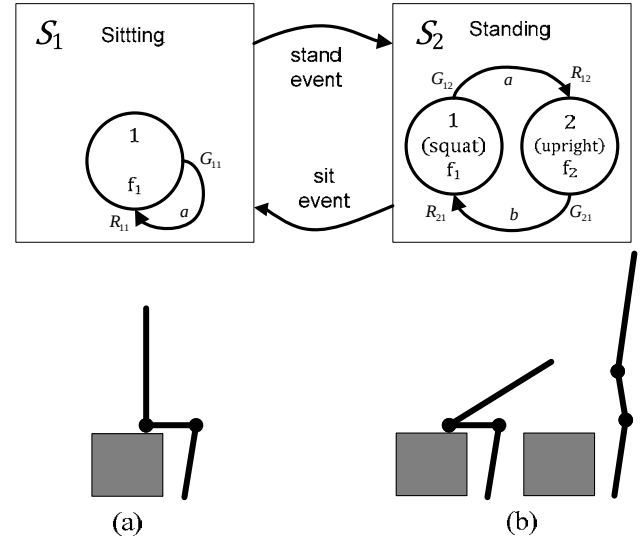


Figure 2. HDS model of a biped sitting to standing

Hybrid dynamic subsystems S_1 and S_2 define sitting and standing respectively. Thus

$$S_1 : \begin{cases} Q : Q = \{q_1 = \text{sitting}\} \\ X : X \in \mathbb{R}^{2n}, X = [x_1, x_2] \\ D : D(q_1) = \left\{ X \in \mathbb{R}^2, x_1 \in \left[\frac{3\pi}{2}, \frac{\pi}{2} \right], x_2 \in [0, \omega_{H \max}] \right\} \\ f : f(q_1, X) = [\dot{X}] = f_1 \\ E : \{(q_1, q_1)\} \\ G : G_{11} = [\text{!s tan d_event}] \\ R : R_{11} = [X^+ = X] \end{cases}$$

Where x_1 and x_2 are the angular position and angular velocity of the hip, $\omega_{H \max}$ is the maximum angular velocity of the hip, and f_1 are the non-linear equations defining the 1-link torso.

Likewise, the hybrid automaton model S_2 is defined as:



$$\begin{aligned}
Q: Q &= \{q_1 = \text{sitting}, q_2 = \text{standing}\} \\
X: X_1 &\in \mathbb{R}^{2n_1}, X_1 = [x_1, x_2], n_1 = 1 \\
X_2 &\in \mathbb{R}^{2n_2}, X_2 = [x_1, x_2, x_3, x_4], n_2 = 2 \\
D: D(q_1) &= \left\{ X_1 \in \mathbb{R}^2, x_1 \in \left[\frac{3\pi}{2}, \frac{\pi}{2} \right], x_2 \in [0, \omega_{H \max}] \right\} \\
D(q_2) &= \left\{ X_2 \in \mathbb{R}^4, x_1 \in \left[\frac{3\pi}{2}, \frac{\pi}{2} \right], x_2 \in [0, \omega_{H \max}], x_3 \in \left[\frac{3\pi}{2}, \frac{\pi}{2} \right], x_4 \in [0, \omega_{K \max}] \right\} \\
f: f(q_1, X_1) &= [\dot{X}_1] = f_1 \\
f(q_2, X_2) &= [\dot{X}_2] = f_2 \\
E &= \{(q_1, q_2), (q_2, q_1)\} \\
G: G_{12} &= [x_1 > \theta_{H12}] \\
G_{21} &= [x_1 < \theta_{H21}, x_3 < \theta_{K21}] \\
R: R_{12} &= \begin{bmatrix} 0 \\ 0 \\ X_{11} \\ X_{12} \end{bmatrix} \\
R_{21} &= \begin{bmatrix} X_{23} \\ X_{24} \end{bmatrix}
\end{aligned} \quad (2)$$

Where for states X_1 , x_1 and x_2 are the angular position and angular velocity of the hip respectively. For states X_2 , x_1 and x_3 are the angular positions of the knee and hip respectively, x_2 and x_4 are the angular velocity of the knee and hip respectively and $\omega_{K \max}$ is the maximum angular velocity of the knee. The nonlinear equations in f_2 define the dynamics of the 2-link torso and knee. Guard conditions are denoted as θ_{H12} , the hip angle threshold to transition to a standing position, and θ_{H21} and θ_{K21} , the hip and knee angle thresholds to be below in order to transition back to a sitting position. During a reset, the mapping R defines how states are assigned. Where X_{11} and X_{12} are the x_1 and x_2 states in the f_1 dynamics. Similarly, X_{23} and X_{24} are the x_3 and x_4 states in the f_2 dynamics.

3. Biped Robot Model Derivation

It is well understood that to simulate walking, a biped robot model needs five degrees of freedom [3]. Several models exist for five link bipeds [1]-[4] and the dynamic models while complicated can be described by nonlinear differential equations. A general equation describing the dynamic motion of a rigid robot takes the form in (3).

$$D(q)\ddot{q} + H(q, \dot{q})\dot{q} + G(q) = u \quad (3)$$

$D(q)$ - the inertia matrix of the robot, size of $n \times n$.

$H(q, \dot{q})$ - the friction, coriolis centripetal forces, size of $n \times 1$.

$G(q)$ - the gravitational forces acting on each link's center of gravity, size of $n \times 1$.

u - the actuator torques acting on each link, size of $n \times 1$.

The differential equations can be derived using Lagrange's method for rigid bodies [5]. The equations of a system are shown in (4) modified from Zaher [6]. The HDS system S_1 in Figure 2(a) represents a sitting biped and thus only the dynamics of the torso are modeled.

Figure 3 represents the single link torso model. For clarity the leg is shown but not annotated.

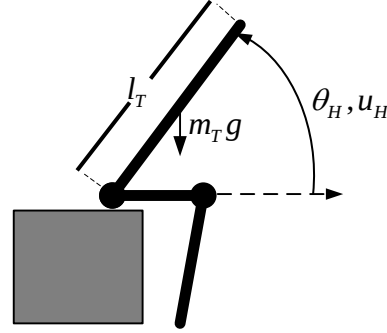


Figure 3. Free body diagram of the single link torso model

$$\frac{1}{3} m_T l_T^2 \ddot{\theta}_H + d_H \dot{\theta}_H + \frac{1}{2} m_T g l_T \sin \theta_H = u_H \quad (4)$$

Where m_T is the mass of the torso, l_T is the length of the torso, d_H is the damping coefficient of the hip joint, g is the acceleration due to gravity and $\theta_H, \dot{\theta}_H, \ddot{\theta}_H$ are the angular position, velocity and acceleration of the torso respectively. The input u_H is the torque applied at the hip. A state space form of (4) can be shown with states $x_1 = \theta_H$ and $x_2 = \dot{\theta}_H$. The state space representation is used in section 4 to design a backstepping controller. Equation (5) also represents the dynamics f_1 in Figure 2(a) in HDS S_1 and Figure 2(b) in HDS S_2 .

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -a \sin x_1 - b x_2 + c u
\end{aligned} \quad (5)$$

$$\text{Where } a = \frac{3g}{2l_T}, b = \frac{3d_H}{m_T l_T^2} \text{ and } c = \frac{3}{m_T l_T^2}$$

The HDS system S_2 in Figure 2(b) represents a biped beginning to rise from the sitting position to an upright standing position. Both right and left legs and feet are kept together. Figure 4 shows the double link model of the torso and upper leg. As before, the derivation of the dynamics is done using Lagrange's method. The equations of this system are shown in equations (6)-(12) which are modified from Zaher [6].

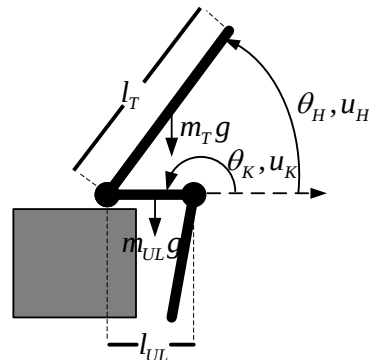


Figure 4. Free body diagram of the two link torso and upper leg model



$$\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_K \\ \ddot{\theta}_H \end{bmatrix} + H \begin{bmatrix} 2\dot{\theta}_K \dot{\theta}_H + \dot{\theta}_H^2 \\ 2\dot{\theta}_K^2 \end{bmatrix} + \begin{bmatrix} G_K \\ G_H \end{bmatrix} = \begin{bmatrix} u_K \\ u_H \end{bmatrix} \quad (6)$$

Where:

$$\begin{aligned} d_{11} &= I_{UL} + I_T + (m_T + m_{UL})(l_{UL}^2 + 2l_{UL}l_{cT} \cos \theta_H) \\ d_{12} &= d_{21} = I_T + (m_T + m_{UL})l_{UL}l_{cT} \cos \theta_H \end{aligned} \quad (7)$$

$$d_{22} = I_T$$

$$l_{cUL} = \frac{1}{2}l_{UL}, l_{cT} = \frac{1}{2}l_T \left(\frac{m_T + 2m_{UL}}{m_T + m_{UL}} \right) \quad (8)$$

$$I_{UL} = \frac{1}{3}m_{UL}l_{UL}^2, I_T = \frac{1}{3}m_Tl_T^2 + m_{UL}l_T^2 \quad (9)$$

$$H = (m_T + m_{UL})l_{UL}l_{cT} \sin \theta_H \quad (10)$$

$$G_H = (m_T + m_{UL})gl_{cT} \sin(\theta_K + \theta_H) \quad (11)$$

$$G_K = G_H + m_{UL}gl_{cUL} \sin \theta_H + (m_T + m_{UL})gl_{UL} \sin \theta_K \quad (12)$$

Where $m_{UL} = 1.5\text{kg}$, $m_T = 1.5\text{kg}$, $l_{UL} = 0.3\text{m}$ and $l_T = 0.2\text{m}$. Where m_{UL} is the mass of the upper leg, l_{UL} is the length of the upper leg, d_K is the damping coefficient of the knee and $\theta_K, \dot{\theta}_K, \ddot{\theta}_K$ are the angular position, velocity and acceleration of the knee respectively. The input u_K is the torque applied at the knee. A state space form of (6) can be shown with states $x_1 = \theta_K$, $x_2 = \dot{\theta}_K$, $x_3 = \theta_H$, $x_4 = \dot{\theta}_H$, $u_1 = u_K$ and $u_2 = u_H$. The state space representation is used in section 4 to design a backstepping controller and is also the representation of f_2 in Figure 2(b) in HDS S_2 .

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ d_{11}\dot{x}_3 + d_{12}\dot{x}_4 + c_{121}x_3x_4 + c_{211}x_4x_3 + c_{221}x_4^2 + G_1 \\ x_4 \\ d_{21}\dot{x}_3 + d_{22}\dot{x}_4 + c_{112}x_3^2 + G_2 \end{bmatrix} + \begin{bmatrix} 0 \\ u_1 \\ 0 \\ u_2 \end{bmatrix} \quad (13)$$

Where the Christoffel symbols, terms c_{ijk} in equation(13), are determined from the following equations from [5].

$$c_{ijk} = \left\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right\} \quad (14)$$

4. Control In Hybrid States

Nonlinear backstepping control is used around each hybrid state. This method of control involves designing virtual stabilizing control functions for each state by using Lyapunov functions [18]. Stability and performance are considered during design with the choice of both the Lyapunov function candidates and the control parameters of the stabilizing functions. Stability of the overall system is guaranteed since the design results in a decomposed sequence of stable Lyapunov functions defining the overall system.

The dynamics of hybrid system S_1 can be re-written from (5) in the following form:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x_1, x_2) + gu \end{aligned} \quad (15)$$

Where $f(x_1, x_2) = -a \sin x_1 - bx_2$ is a typical nonlinear equation describing the 1-link system. The first step of the backstepping design is to stabilize the $\dot{x}_1$ substate dynamics with a stabilizing function, $\alpha_1(\cdot)$.

We augment the design by adding integral action to help reject disturbances and reduce steady state error. This design method is similar to the work in [11]. A new state variable $z_1 = x_{1d} - x_1$ is introduced where x_{1d} is the desired hip angle to track. By taking time derivative:

$$\dot{z}_1 = \dot{x}_{1d} - x_2 \quad (16)$$

An error variable z_2 is then defined.

$$z_2 = \alpha_1(\cdot) - x_2 \quad (17)$$

Substituting (17) into (16) gives the dynamics of the new state $\dot{z}_1$ in terms of z_2 :

$$\dot{z}_1 = \dot{x}_{1d} - \alpha_1(\cdot) + z_2 \quad (18)$$

Next a control Lyapunov function (clf) is used to determine $\alpha_1(\cdot)$. The following clf is considered:

$$V_1 = \frac{k_1}{2}\zeta^2 + \frac{1}{2}z_1^2 \quad (19)$$

The first term takes into account the integral of the tracking error,

$$\zeta = \int_0^t z_1(\tau) d\tau \quad (20)$$

Taking the time derivative of (19) results in:

$$\begin{aligned} \dot{V}_1 &= k_1\zeta \dot{z}_1 + z_1\dot{z}_1 \\ &= z_1(k_1\zeta + \dot{x}_{1d} - \alpha_1(\cdot) + z_2) \end{aligned} \quad (21)$$

For the state z_1 to be stable, $\alpha_1(\cdot)$ must be defined to

make $\dot{V}_1 < 0$. Consider the following possible

stabilizing function where c_1 and k_1 are both positive control parameters:

$$\alpha_1(\cdot) = c_1z_1 + \dot{x}_{1d} + k_1\zeta \quad (22)$$

Taking the time derivative of (22) results in:

$$\dot{\alpha}_1(\cdot) = c_1\dot{z}_1 + \ddot{x}_{1d} + k_1\dot{\zeta} \quad (23)$$

Substituting (22) into (21) results in:

$$\begin{aligned} \dot{V}_1 &= z_1(k_1\zeta + \dot{x}_{1d} - (c_1z_1 + \dot{x}_{1d} + k_1\zeta) + z_2) \\ &= z_1(-c_1z_1 + z_2) \\ &= -c_1z_1^2 + z_1z_2 \end{aligned} \quad (24)$$

The state z_1 is asymptotically stable when the error z_2 is zero, $\dot{V}_1 < 0$.



The second step is to stabilize the $\dot{x}_2$ sub state dynamics using a similar procedure that was used in step 1. Differentiating (17) and from state equations, results in the dynamics of the error variable z_2 :

$$\begin{aligned}\dot{z}_2 &= \dot{\alpha}_1(\cdot) - \dot{x}_2 \\ &= \dot{\alpha}_1(\cdot) - f(x_1, x_2) - gu\end{aligned}\quad (25)$$

The following clf is considered to stabilize the second state:

$$V_2 = V_1 + \frac{1}{2} z_2^2 \quad (26)$$

Taking the time derivative of (26) results in:

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + z_2 \dot{z}_2 \\ &= -c_1 z_1^2 + z_1 z_2 + z_2 (\dot{\alpha}_1(\cdot) - f(x_1, x_2) - gu)\end{aligned}\quad (27)$$

Since the control input u is part of the error dynamics no additional stabilizing function is required. Instead, u is defined to stabilize the $\dot{z}_2$ state dynamics by making $\dot{V}_2 < 0$. This is done by introducing another control parameter, c_2 , and by defining the desired error dynamics.

$$\dot{z}_2 = -c_2 z_2 - z_1 \quad (28)$$

Then by using (25) and (28) the control input can be determined.

$$\begin{aligned}\dot{\alpha}_1(\cdot) - f(x_1, x_2) - gu &= -c_2 z_2 - z_1 \\ u &= \frac{1}{g} (\dot{\alpha}_1(\cdot) - f(x_1, x_2) + c_2 z_2 + z_1) \\ u &= \frac{1}{c} (a \sin x_1 + bx_2 + \ddot{x}_{1d} + (k_1 + 1 - c_1^2) z_1 - c_1 k_1 \zeta + (c_1 + c_2) z_2)\end{aligned}\quad (29)$$

Substituting (29) back into (27) results in:

$$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2 \quad (30)$$

When c_2 is a positive control parameter $\dot{V}_2 < 0$.

Control parameters c_1 and c_2 are set to adjust the desired behavior of the asymptotically decreasing position and velocity errors respectively. The parameter k_1 sets the gain on the added effect resulting from integral action. A diagram of the original system of equations (15) is shown in Figure 5. The additions to the system diagram with backstepping and integral action are shown in Figure 6. The control function u for the backstepping controller is shown in Figure 7.

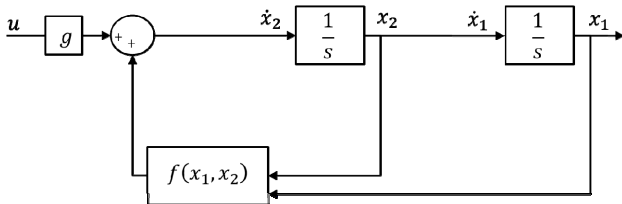


Figure 5. System diagram of the original system S_1

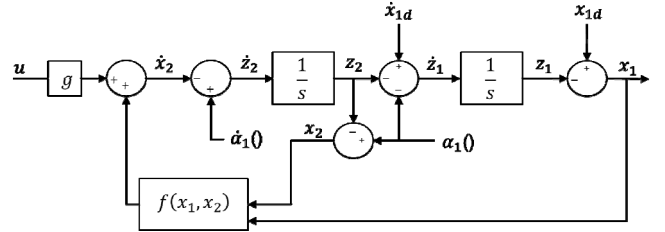


Figure 6. System diagram showing backstepping with integral action control for system S_1

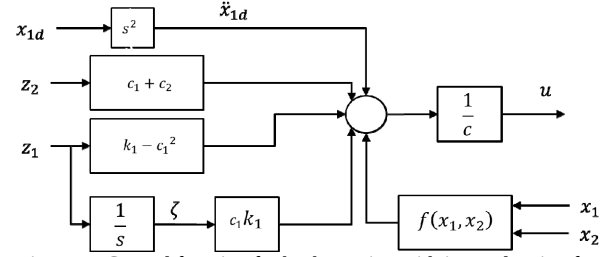


Figure 7. Control function for backstepping with integral action for system S_1

The dynamics for f_2 in the upright state of HDS S_2 can be rewritten from (13) in the following form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ f_k(\cdot) \\ x_4 \\ f_h(\cdot) \end{bmatrix} + \begin{bmatrix} 0 \\ u_1 \\ 0 \\ u_2 \end{bmatrix} \quad (31)$$

Where $f_k(\cdot)$ and $f_h(\cdot)$ are nonlinear equations describing the knee and hip dynamics respectively. The first two steps of the backstepping design follow from the single link example with some changes to the variable subscripts to allow for additional states and control inputs. A new state variable $z_1 = x_{1d} - x_1$ is introduced where x_{1d} is the desired knee angle to track. By taking time derivative:

$$\dot{z}_1 = \dot{x}_{1d} - \dot{x}_2 \quad (32)$$

An error variable z_2 is then defined.

$$z_2 = \alpha_1(\cdot) - x_2 \quad (33)$$

Substituting (33) into (32) gives the dynamics of the new state $\dot{z}_1$ in terms of z_2 :

$$\dot{z}_1 = \dot{x}_{1d} - \alpha_1(\cdot) + z_2 \quad (34)$$

Next a control Lyapunov function (clf) is used to determine $\alpha_1(\cdot)$. The following clf is considered:

$$V_1 = \frac{k_1}{2} \zeta_1^2 + \frac{1}{2} z_1^2 \quad (35)$$

The first term takes into account the integral of the tracking error,

$$\zeta_1 = \int_0^t z_1(\tau) d\tau \quad (36)$$

Taking the time derivative of (35) results in:



$$\begin{aligned}\dot{V}_1 &= k_1 \zeta_1 z_1 + z_1 \dot{z}_1 \\ &= z_1 (k_1 \zeta_1 + \dot{x}_{1d} - \alpha_1(\cdot) + z_2)\end{aligned}\quad (37)$$

For the state z_1 to be stable, $\alpha_1(\cdot)$ must be defined to make $\dot{V}_1 < 0$. Consider the following possible stabilizing function where c_1 and k_1 are both positive control parameters:

$$\alpha_1(\cdot) = c_1 z_1 + \dot{x}_{1d} + k_1 \zeta_1 \quad (38)$$

Taking the time derivative of (38) results in:

$$\dot{\alpha}_1(\cdot) = c_1 \dot{z}_1 + \ddot{x}_{1d} + k_1 \dot{z}_1 \quad (39)$$

Substituting (38) into (37) results in:

$$\begin{aligned}\dot{V}_1 &= z_1 (k_1 \zeta_1 + \dot{x}_{1d} - (c_1 z_1 + \dot{x}_{1d} + k_1 \zeta_1) + z_2) \\ &= z_1 (-c_1 z_1 + z_2) \\ &= -c_1 z_1^2 + z_1 z_2\end{aligned}\quad (40)$$

The state z_1 is asymptotically stable when the error z_2 is zero, $\dot{V}_1 < 0$.

The second step is to stabilize the $\dot{x}_2$ sub state dynamics using a similar procedure that was used in step 1. Differentiating (33) and from state equations, results in the dynamics of the error variable z_2 :

$$\begin{aligned}\dot{z}_2 &= \dot{\alpha}_1(\cdot) - \dot{x}_2 \\ &= \dot{\alpha}_1(\cdot) - f_k(\cdot) - u_1\end{aligned}\quad (41)$$

The following clf is considered to stabilize the second state:

$$V_2 = V_1 + \frac{1}{2} z_2^2 \quad (42)$$

Taking the time derivative of (42) results in:

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + z_2 \dot{z}_2 \\ &= -c_1 z_1^2 + z_1 z_2 + z_2 (\dot{\alpha}_1(\cdot) - f_k(\cdot) - u_1)\end{aligned}\quad (43)$$

Since the control input u_1 is part of the error dynamics no additional stabilizing function is required. Instead, u_1 is defined to stabilize the $\dot{z}_2$ state dynamics by making $\dot{V}_2 < 0$. This is done by introducing another control parameter, c_2 and by defining the desired error dynamics.

$$\dot{z}_2 = -c_2 z_2 - z_1 \quad (44)$$

Then by using (41) and (44) the control input can be determined.

$$\begin{aligned}\dot{\alpha}_1(\cdot) - f_k(\cdot) - u_1 &= -c_2 z_2 - z_1 \\ u_1 &= -f_k(\cdot) + \dot{\alpha}_1(\cdot) + c_2 z_2 + z_1\end{aligned}\quad (45)$$

Substituting (45) back into (43) results in:

$$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2 \quad (46)$$

When c_2 is a positive control parameter $\dot{V}_2 < 0$.

The following two steps design the backstepping control for the states describing the hip dynamics. A new state

variable $z_3 = x_{3d} - x_3$ is introduced where x_{3d} is the desired hip angle to track. By taking time derivative:

$$\dot{z}_3 = \dot{x}_{3d} - \dot{x}_3 \quad (47)$$

An error variable z_4 is then defined.

$$z_4 = \alpha_2(\cdot) - x_4 \quad (48)$$

Substituting (48) into (47) gives the dynamics of the new state $\dot{z}_3$ in terms of z_3 :

$$\dot{z}_3 = \dot{x}_{3d} - \alpha_2(\cdot) + z_4 \quad (49)$$

Next a control Lyapunov function (clf) is used to determine $\alpha_2(\cdot)$. The following clf is considered:

$$V_3 = V_2 + \frac{k_2}{2} \zeta_2^2 + \frac{1}{2} z_3^2 \quad (50)$$

The first term takes into account the integral of the tracking error,

$$\zeta_2 = \int_0^t z_3(\tau) d\tau \quad (51)$$

Taking the time derivative of (50) results in:

$$\begin{aligned}\dot{V}_3 &= \dot{V}_2 + k_2 \zeta_2 z_3 + z_3 \dot{z}_3 \\ &= -c_1 z_1^2 - c_2 z_2^2 + z_3 (k_2 \zeta_2 + \dot{x}_{3d} - \alpha_2(\cdot) + z_4)\end{aligned}\quad (52)$$

For the state z_3 to be stable, $\alpha_2(\cdot)$ must be defined to make $\dot{V}_3 < 0$. Consider the following possible stabilizing function where c_3 and k_2 are both positive control parameters:

$$\alpha_2(\cdot) = c_3 z_3 + \dot{x}_{3d} + k_2 \zeta_2 \quad (53)$$

Taking the time derivative of (53) results in:

$$\dot{\alpha}_2(\cdot) = c_3 \dot{z}_3 + \ddot{x}_{3d} + k_2 \dot{z}_3 \quad (54)$$

Substituting (53) into (52) results in:

$$\begin{aligned}\dot{V}_3 &= -c_1 z_1^2 - c_2 z_2^2 + z_3 (k_2 \zeta_2 + \dot{x}_{3d} - (c_3 z_3 + \dot{x}_{3d} + k_2 \zeta_2) + z_4) \\ &= -c_1 z_1^2 - c_2 z_2^2 + z_3 (-c_3 z_3 + z_4) \\ &= -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 + z_3 z_4\end{aligned}\quad (55)$$

The state z_3 is asymptotically stable when the error z_4 is zero, $\dot{V}_3 < 0$.

The final step is to stabilize the $\dot{x}_4$ sub state dynamics using a similar procedure that was used in step 2. Differentiating (48) and from state equations, results in the dynamics of the error variable z_4 :

$$\begin{aligned}\dot{z}_4 &= \dot{\alpha}_2(\cdot) - \dot{x}_4 \\ &= \dot{\alpha}_2(\cdot) - f_h(\cdot) - u_2\end{aligned}\quad (56)$$

The following clf is considered to stabilize the second state:

$$V_4 = V_3 + \frac{1}{2} z_4^2 \quad (57)$$

Taking the time derivative of (57) results in:



$$\begin{aligned}\dot{V}_4 &= \dot{V}_3 + z_4 \dot{z}_4 \\ &= -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 + z_3 z_4 + z_4 (\dot{\alpha}_2(\cdot) - f_h(\cdot) - u_2)\end{aligned}\quad (58)$$

Since the control input u_2 is part of the error dynamics no additional stabilizing function is required. Instead, u_2 is defined to stabilize the $\dot{z}_4$ state dynamics by making $\dot{V}_4 < 0$. This is done by introducing another control parameter, c_4 and by defining the desired error dynamics.

$$\dot{z}_4 = -c_4 z_4 - z_3 \quad (59)$$

Then by using (56) and (59) the control input can be determined.

$$\begin{aligned}\dot{\alpha}_2(\cdot) - f_h(\cdot) - u_2 &= -c_4 z_4 - z_3 \\ u_2 &= -f_h(\cdot) + \dot{\alpha}_2(\cdot) + c_4 z_4 + z_3\end{aligned}\quad (60)$$

Substituting (60) back into (58) results in:

$$\dot{V}_4 = -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 - c_4 z_4^2 \quad (61)$$

When c_4 is a positive control parameter $\dot{V}_4 < 0$.

Control parameters c_1 and c_2 are set to adjust the desired behavior of the asymptotically decreasing position and velocity errors respectively of the knee. Likewise the control parameters c_3 and c_4 are for the hip. The parameters k_1 and k_2 set the gain on the added effect resulting from integral action for the knee and hip respectively.

5. Simulations

Sitting to standing posture simulation is performed with the sitting portion over a fixed period of time followed by a standing portion with another fixed period of time. Each fixed portion of time represents the time in HDS state S_1 and S_2 with a standing event triggered between the fixed time intervals. During each HDS state a disturbance is introduced to examine robustness. This disturbance represents a force applied on the biped torso during state S_1 and applied to the upper leg during state S_2 . Robustness to system parameter variations and the benefits of integral action are also demonstrated.

The reference positions are pre-filtered through a model reference to provide the positions and derivatives required by the backstepping controller. A second order reference model is used to provide realistic trajectories to track without overshoots [5]. The use of a second-order model also makes it possible to obtain the velocity and acceleration information from the reference without the need to specify these trajectories. The following model was used from Khalil [18].

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} y_2 \\ -\omega_n^2 y_1 - 2\zeta\omega_n y_2 + \omega_n^2 x_{1d} \end{bmatrix} \quad (62)$$

Parameter of the model are set to $\omega_n = 10$ and $\zeta = 1$ to give a critically damped response with approximately a 0.5s settling time.

Initial positions for S_1 are $\theta_H(0) = 0$,

$\theta_K(0) = -\frac{\pi}{2}$ which simulates a sitting position with the torso completely vertical. The desired position for S_1 is $\theta_{H_{des}} = \frac{\pi}{6}$ which is a slightly leaned forward torso.

Once the transition from state S_1 to S_2 occurs the desired torso position changes to $\theta_{H_{des}} = \frac{\pi}{3}$ which is a deeply

leaned forward position to get the torso center of gravity above the position of the foot. This position is in preparation to stand upright. Once S_2 has transitioned to the upright sub-state, f_2 , the desired positions change to the final upright standing posture of $\theta_{H_{des}} = 0$, $\theta_{K_{des}} = 0$.

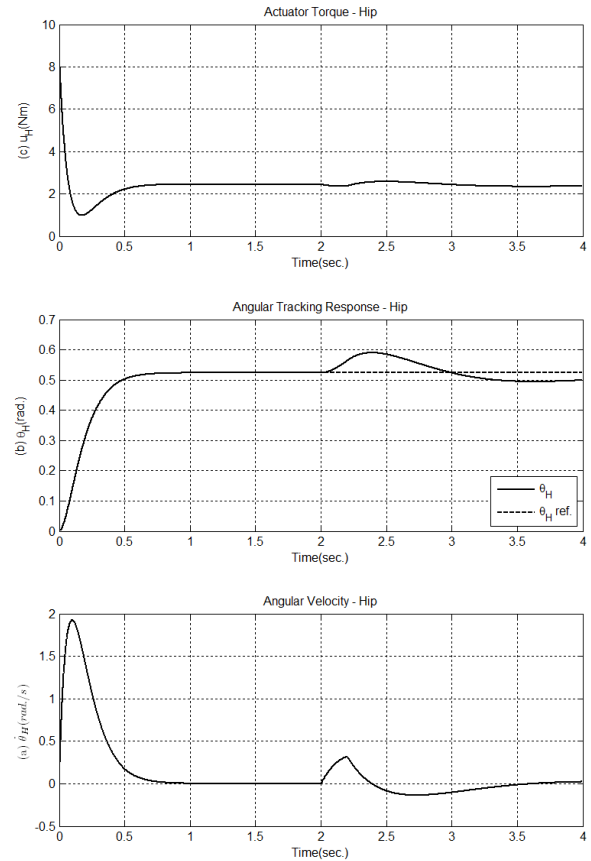


Figure 8. 1-link backstepping response for HDS S_1

Figure 8. shows the response during the state S_1 . The actuator torque, desired and actual positions, angular velocity and a phase plane plot of θ_H vs. $\dot{\theta}_H$ is also shown. From the angular tracking response plot is seen that the model reference trajectory is followed closely.



At $t = 2s$ a 3Nm disturbance torque is applied for 200ms. The resulting correction in tracking can be seen as the loop in the phase plot.

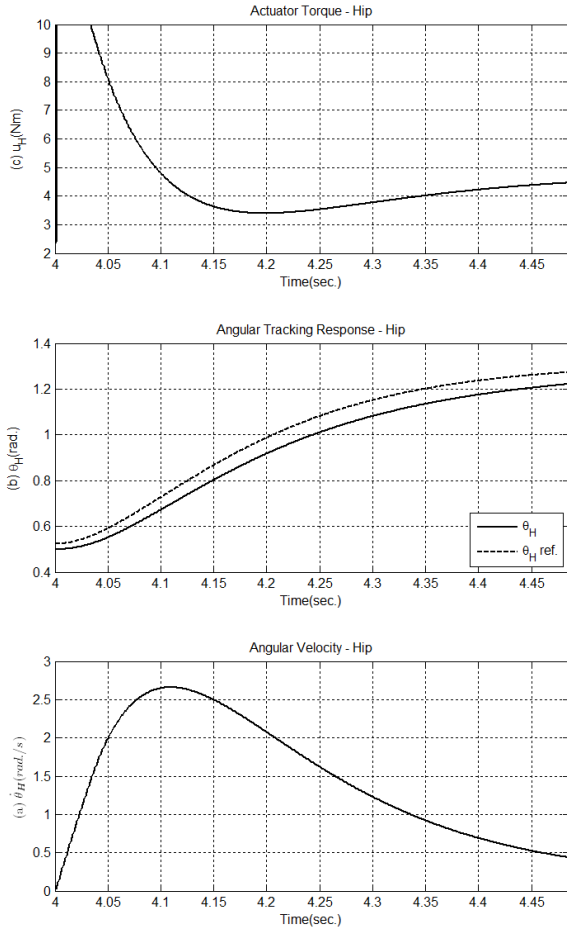


Figure 9. 1-link backstepping response for HDS $S_2 f_1$

Figure 9. shows the response during the state S_2 when the dynamics f_1 are present. This is the hip only portion of motion where the desired position is a deep lean forward. From the angular tracking response plot it is seen that the model reference trajectory is followed by the hip.

Figure 12. shows the total system response with four second fixed periods of time in states S_1 and S_2 . A standing event is simulated at $t = 4s$. Disturbance pulses of 3Nm are present for 200ms at $t = 2s$ on the torso and at $t = 6s$ on the upper leg. The reference and actual position of the hip and knee along with actuator torque is shown. Notice that during state S_1 the knee is assumed fixed in position since only the torso is moving on the seated biped. No actuator torque is applied to the knee until in state S_2 when in the upright sub-state, q_2 of Figure 2(b).

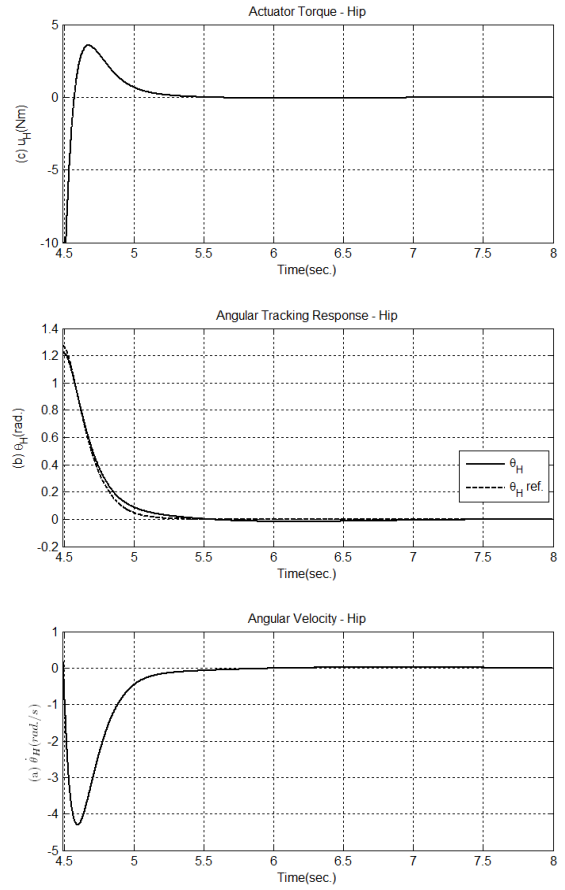


Figure 10. 2-link backstepping response of the hip for HDS $S_2 f_2$

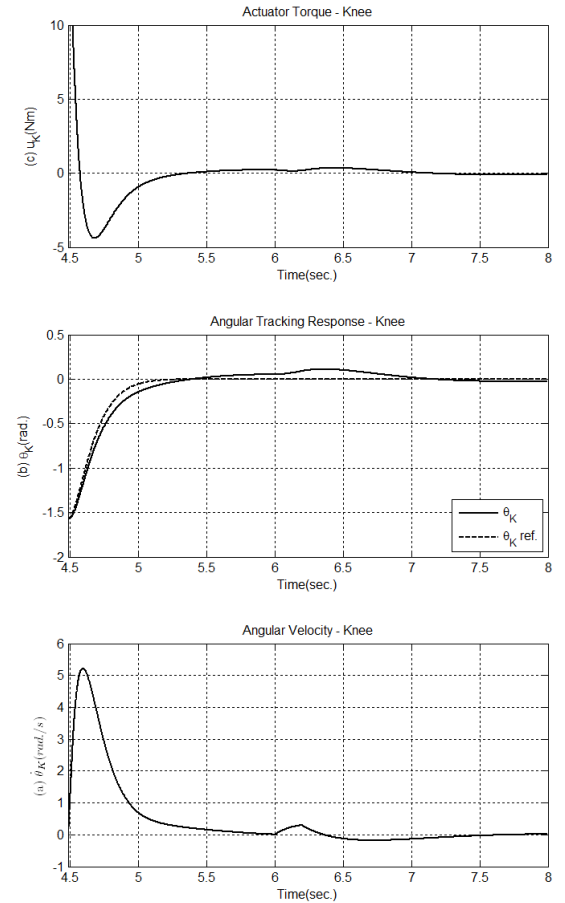


Figure 11. 2-link backstepping response of the knee for HDS $S_2 f_2$



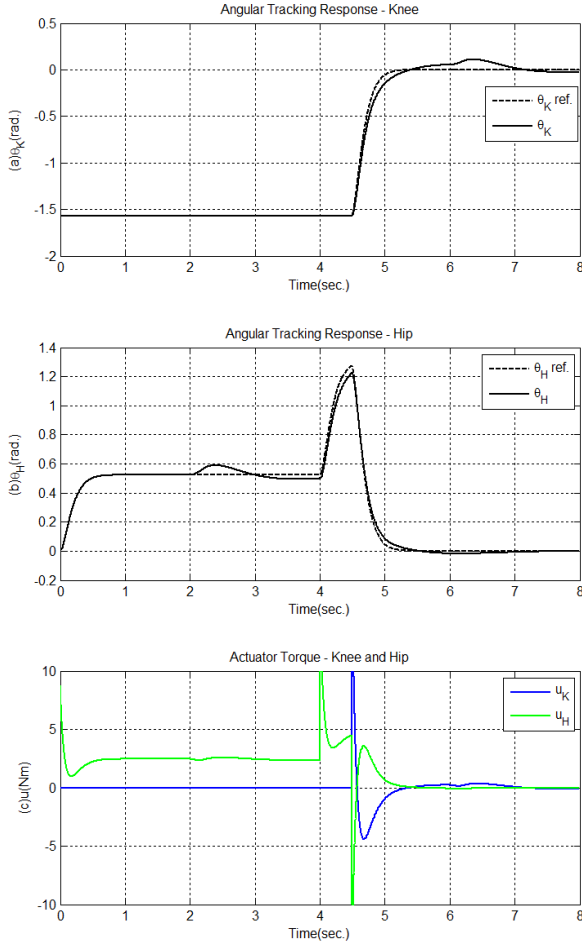


Figure 12. Total system response over HDS S_1 to S_2 using backstepping control

This simulation uses the following controller parameters $c_1, c_3, c_3, c_4 = 3, k_1, k_2 = 4, a = 29.43, b = 3, c = 6$

Figure 13. shows the same system simulated with PD control instead of using the backstepping control from section 4. The backstepping control is much smoother in following the references trajectories when compared to the PD control. Additionally, there is less output chattering and lower output magnitudes required when using the backstepping control.

Simulink and Simmechanics diagrams of the system are shown in figure 14 and figure 15 respectively. Simmechanics was established to be a representative method to simulate robotic systems by Dung and Kang [14]. The authors have also shown the use of Simmechanics in simulations of hybrid dynamic systems in their previous work [19].

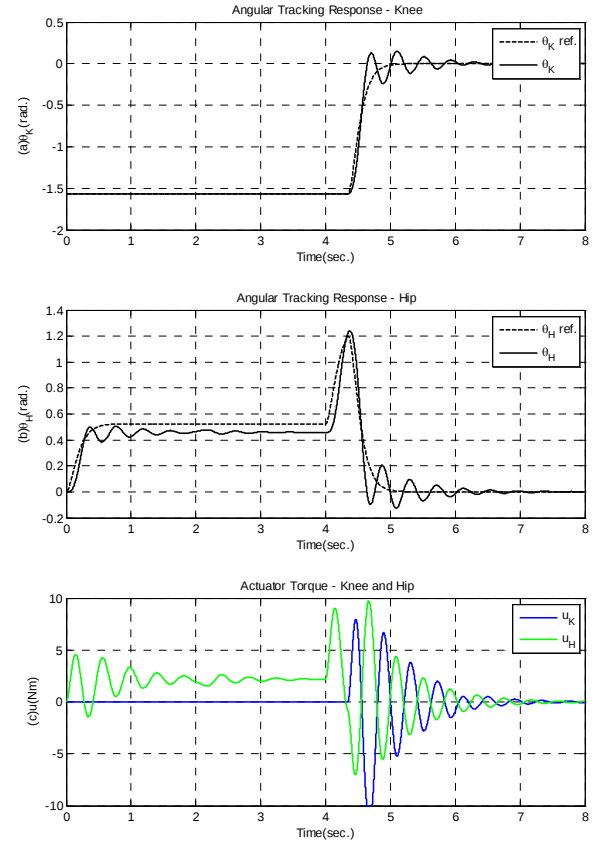


Figure 13. Total system response over HDS S_1 to S_2 using PD control

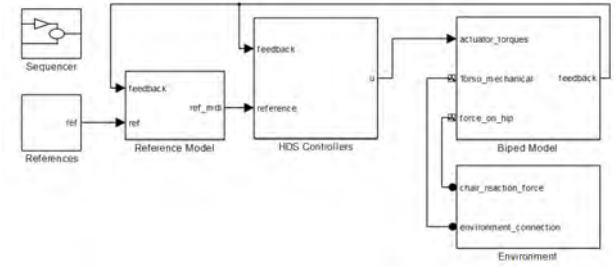


Figure 14. Simulink diagram of the simulated system

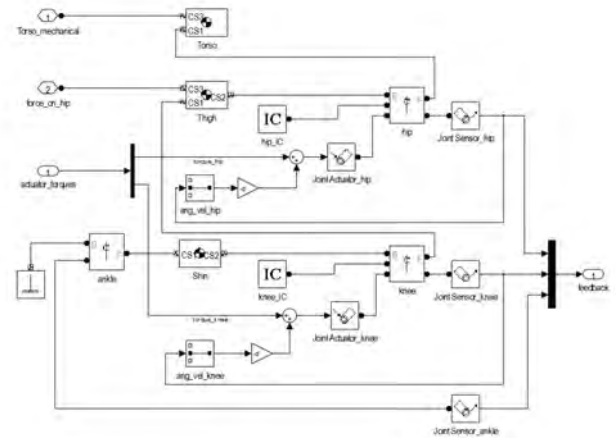


Figure 15. Simmechanics model of biped



5.1 Robustness

Simulations to demonstrate robustness are divided into two parts. First, the robustness to parameter variations of the system is examined. Second, the robustness to disturbance is examined. Parameters for the mass, length and joint damping are changed $\pm 30\%$ to see the effects on the response. Phase plots during state S_1 are shown in the following figures, Figure 16-18, to illustrate the response differences as parameters are adjusted.

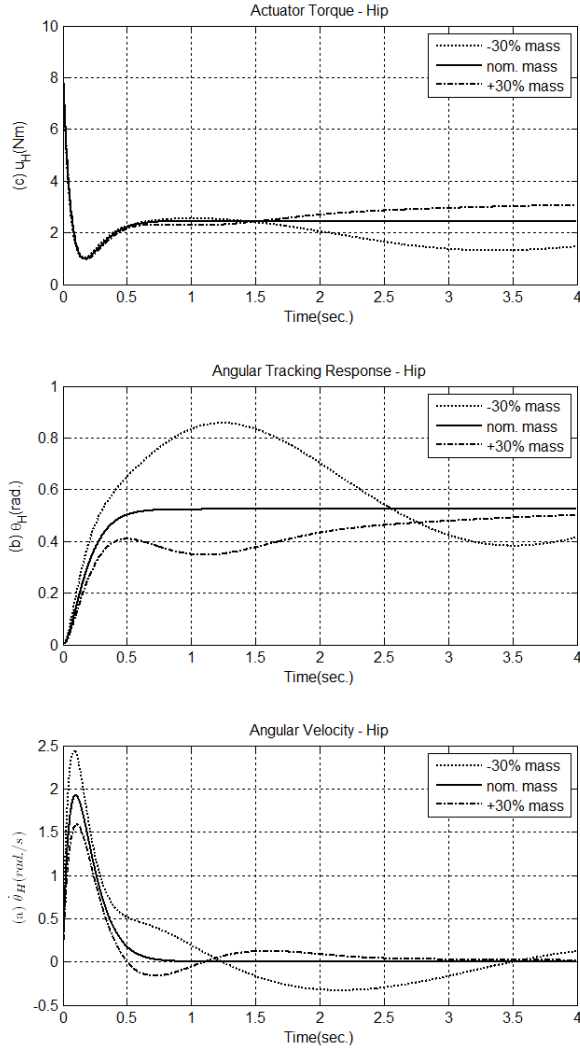


Figure 16. Robustness to variations in system mass

The controller is robust to mass variation in the system. A system with less mass responds quicker and therefore is more difficult to control with the same actuator. However, the response does show that even for the -30% mass variation the system is asymptotically stable.

Robustness as a result of length variation is very similar to mass variations. A shorter link responds quicker and is more difficult to control with a given actuator. Again, the response for the reduced length variation is still asymptotically stable.

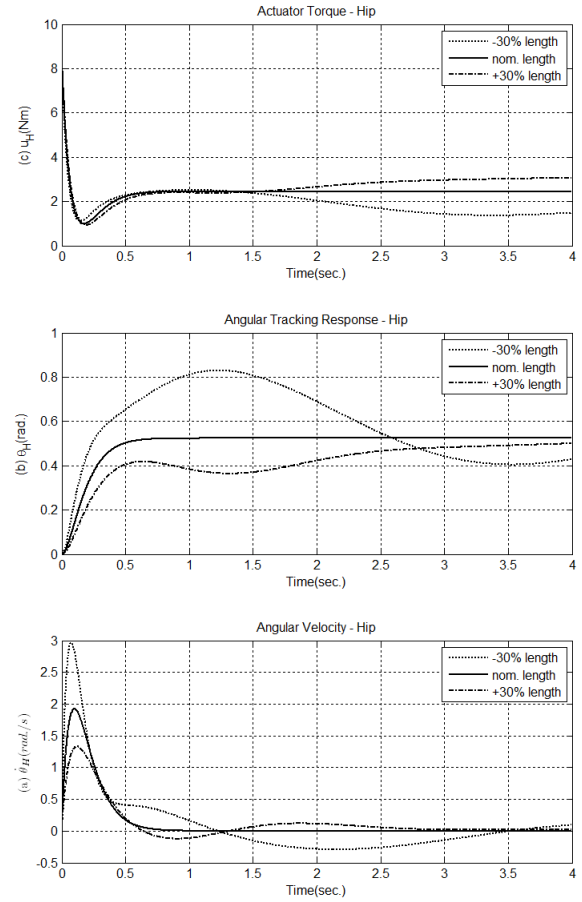


Figure 17. Robustness to variations in system length

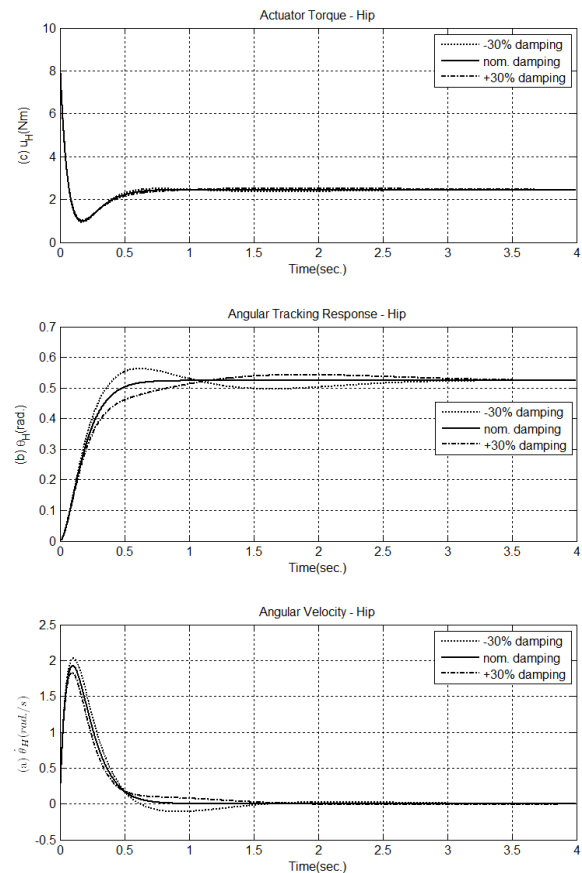


Figure 18. Robustness to variations in system damping



Robustness to damping variation is the best of the three parameters. The effect of damping does not contribute as much to the system response and therefore the controller is able to compensate changes well.

5.2 Disturbance Rejection

Disturbance during the S1 and S2 states is simulated by a step function of applied torque. The simulations are repeated with and without the integral action of the backstepping controller to show the performance difference. The disturbance signal is shown in Figure 19. The amplitude is 3Nm and the duration is 200ms. Trajectory tracking performance is also examined to see the benefits of the integral action.

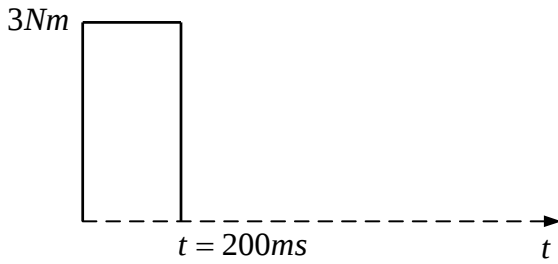


Figure 19. Disturbance signal

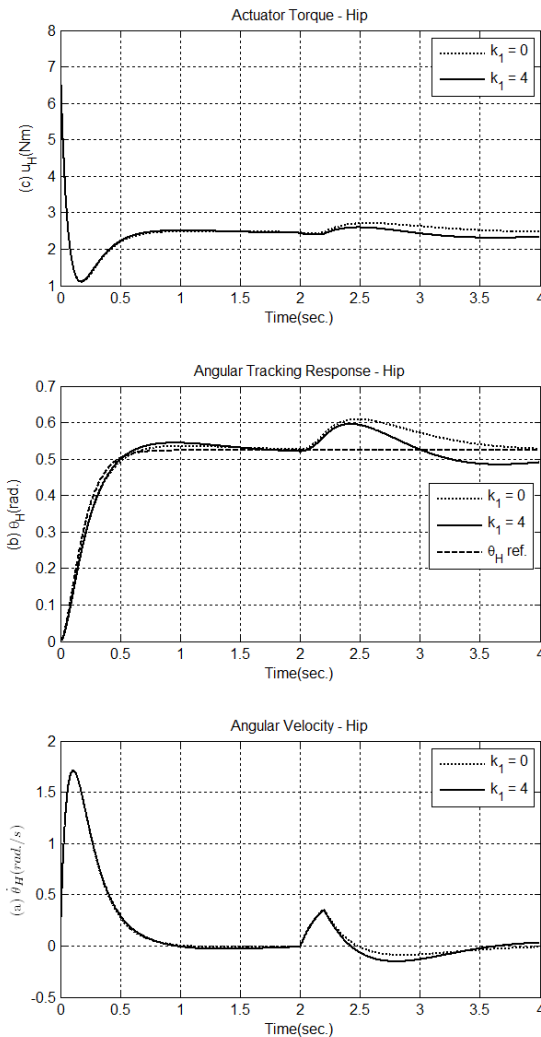


Figure 20. Response comparing disturbance rejection with and without integral action

The integral action improves the response time to reject the disturbance effect with the consequence of adding overshoot. This is seen in Figure 20(b).

Figure 21 examines the trajectory tracking performance of the integral action. A trapezoidal trajectory is used where there is a constant slope from $t = 0s$ to $t = 2s$. Then at $t = 2s$ the reference is at a fixed value of $\frac{\pi}{6}$ radians. Again as in Figure 20(b) the response with integral action is faster with some overshoot. The error and sum squared error is examined further in Figure 22. The error between the responses with and without integral action appears to show that without integral action is better. However, looking at the sum square error the integral action response does perform better during the ramp portion of the trajectory tracking and has an overall smaller sum square error.

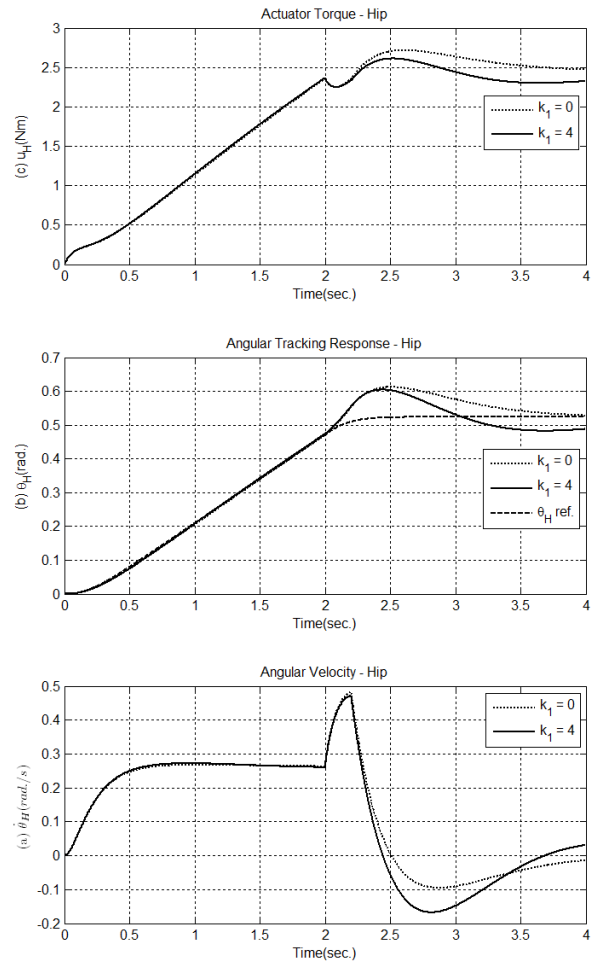


Figure 21. Response comparing trajectory tracking with and without integral action



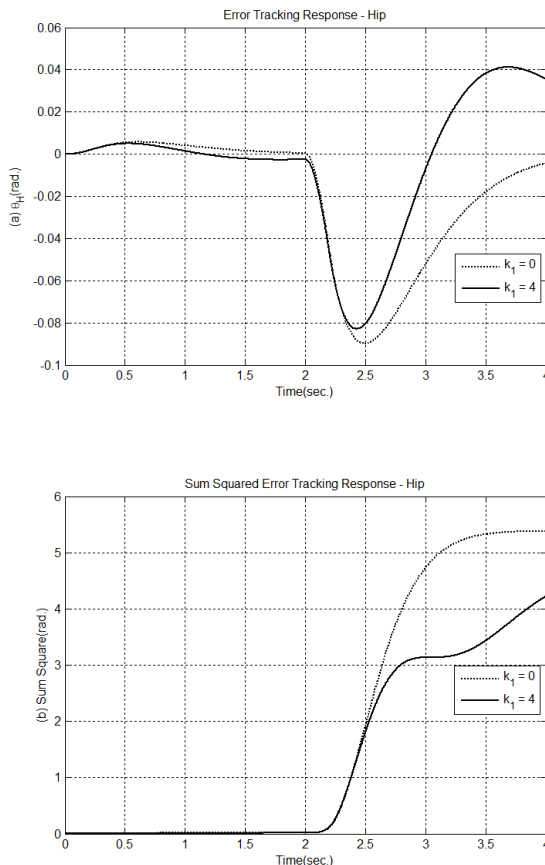


Figure 22. Error and sum squared error of trajectory tracking with and without integral action

6. Conclusion

Sequenced hybrid dynamic systems have been shown to be useful in the control of simulated biped robot motion from a sitting to standing pose. Backstepping control with integral action was applied to control the biped in the individual hybrid states. Robustness of the backstepping control with integral action was shown to be robust to variations in parameters in mass, length and damping with an asymptotically stable response. The benefit of integral action to the backstepping control was shown to improve the response time and results in an overall better performance during trajectory tracking with regards to the sum square error of the response. Overall the control method presented when compared to a classical PD control is better from trajectory tracking and smoother output actuator torque. Extensions to this work with additional sequenced poses and motions are planned.

References

- [1] Y. Hurmuzlu, "Dynamics and Control of Bipedal Robots," in: B. Siciliano, K. Valavanis (Eds.), *Control Problems in Robotics and Automation*, Springer Berlin, Heidelberg, 1998, pp. 105-117.
- [2] E. R. Westervelt, G. Buche, and J. W. Grizzle., "Inducing Dynamically Stable Walking in an Underactuated Prototype Planar Biped," *Proceedings of 2004 IEEE International Conference on Robotics and Automation*, pp. 4234-4239, vol.4., 2004.
- [3] E. R. Westervelt, J. W. Grizzle, C. Chevallereau, J. H. Choi, B. Morris, "Feedback Control of Dynamic Bipedal Robot Locomotion," CRC Press, Boca Raton, FL, 2007.
- [4] Y. Otoda, H. Kimura, "Efficient Walking with Torso using Natural Dynamics," *Mechatronics and Automation*, 2007. ICMA 2007. International Conference on, pp. 1914-1919, Aug. 2007.
- [5] M. W. Spong, M. Vidyasagar., "Robot Dynamics and Control," Wiley, New York, 1989.
- [6] A. A. Zaher, "Novel nonlinear techniques in modeling, simulation, and control with applications to biped robots," Ph.D. thesis, Oakland University, 2001.
- [7] T. Fossen, J. Strand, "Tutorial on Nonlinear Backstepping: Applications to Ship Control," *Modeling, Identification and Control*, vol. 20, no. 2, pp. 83-134, 1999.
- [8] J. Zhou, C. Wen, "Adaptive Backstepping Control of Uncertain Systems," *LNCIS* 372, pp. 9-31, 2008.
- [9] S. Hasan, K. C. Cheok, "Adaptive Backstepping Control Based on Estimation of Dominant Parameters for Brushless DC Motor," *ICGST ASCE Journal*, vol. 12, no. 3, pp. 1-6, December 2012.
- [10] A. Belhani, F. Mehazzem, K. Belarbi, "Decentralized Backstepping Controller for a Class of Nonlinear Multivariable Systems," *ICGST ASCE Journal*, vol. 6, no. 2, pp. 41-48, June 2006.
- [11] Y. Tan, and J. Chang, H. Tan, J. Hu, "Integral Backstepping Control and Experimental Implementation for Motion System," *Proceedings of the 2000 IEEE International Conference on Control Applications*, pp. 367-372, Anchorage, Alaska, September 2000.
- [12] A. M. Harb, A. A. Zaher, A. A. Al-Qaisia, M. A. Zohdy, "Recursive Backstepping Control of Chaotic Duffing Oscillators," *Chaos, Solitons and Fractals*, vol. 34, no. 2, pp. 639-645, Oct. 2007.
- [13] X. Zhao, Y. Liu, "Modeling of Biped Robot," in *Control and Decision Conference (CCDC)*, 2010 Chinese, pp. 3233-3238, May 2010.
- [14] L. T. Dung, H. Kang, Y. Ro, "Robot Manipulator Modeling in Matlab-Simmechanics with PD Control and Online Gravity Compensation," *Strategic Technology (IFOST)*, 2010 International Forum on. (2010) 446-449.
- [15] T. Manrique, D. Patiño, "Mathematical Modeling on Hybrid Dynamical Systems: An Application to the Bouncing Ball and a Two-Tanks System," *ANDESCON, IEEE* (2010) 1-8.
- [16] R. Goebel, R. Sanfelice, A. Teel, "Hybrid Dynamical Systems, Control Systems," *IEEE Control Systems Magazine*, vol. 29.2, pp. 28-93, 2009.
- [17] A. J. van der Schaft, J. M. Schumacher, "Examples of Hybrid Dynamical Systems," *Lecture Notes in Control and Information Sciences*, Springer Berlin, Heidelberg, 2000.
- [18] H. Khalil, "Nonlinear Systems," third ed., Prentice Hall, New Jersey, 2002.
- [19] J. Piasecki, M. A. Zohdy, "Robust Hybrid Complex Motion Control Using Fuzzy Logic, Inverse Dynamic and PID-Q Controllers," *International Review of Automatic Control (IREACO)*, vol. 6, no. 2, pp. 19-28, 2013.



Biographies



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